

*Full Length Research Paper*

# Multifractality nature and multifractal detrended fluctuation analysis of well logging data

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Geophysical well logging data often show a complex pattern due to multifractal condition. Using Multifractal Detrended Fluctuation Analysis on the original, shuffled and surrogated series is a procedure in finding the source of multifractality. Fractal analysis on bulk density, sonic transmit time, acoustic impedance and neutron porosity of two wells from two Iranian oil fields were done. Results respectively showed similar scaling behavior of the long-term correlation and the broad probability distribution in result of the depositional environment and the strong heterogeneity over the time scale and confirmed that different layers cannot be introduced with the same power law exponent.

**Keywords:** Well logging, Self-similarity, Multifractal detrended fluctuation analysis, Shuffled series, Surrogated series, Hurst coefficient.

## INTRODUCTION

Self-affine fractals are characteristics which described many natural events like biology, physics, geology and geophysics (De Santis et al., 1997; Dimri, 2000; Fedi, 2003; Bansal and Dimri, 2005a; Moktadir et al., 2008). Self-similar fractals show a pattern that is similar to itself in any scale, and they are the general form of fractal Brownian motions (fBm) and fractal Gaussian noises (fGn) (Turcotte, 1997; Malamud and Turcotte, 1999). The fBm are defined by Hurst coefficient (H) and show the scaling nature of the motions with the value between 0 and 1. Various methods like the Power Spectrum (PS), Roughness Length (RL), Semi-Variogram (SV), Wavelet Transform (WT), Spectral Density (SD) and Rescaled range (R/S) can estimate Hurst coefficient.

A signal called homogeneous fractal or monofractal, when fBm are singular with the same Holder exponent. Multifractality is introduced by Mandelbrot (1974) to show turbulence phenomena. Later Multifractality Concept has been applied in very different contexts (Mandelbrot, 1989). Multifractal signal models are positive distributions with self-similarity but they have non-homogeneous scaling. Multifractal scaling supplies a quantitative

description of a broad range of heterogeneous events that can be distinguished between different regions, which have different fractal properties (Stanley and Meakin, 1988).

Well logging data represent stochastic spatial series caused by sedimentation and other processes after that. So, the analysis of well logging data is a complex task and the physical property of the earth is the reason of multifractality in well logging data. In a multifractal system, any piece of the system is established by a distinct exponent. So, for the characterization of a system, a large number of such exponents are needed. The crust is formed from many layers with different characteristics and heterogeneities due to sedimentation, compaction, cementation, stratification, tectonic activity and so forth. So, one can expect different layers represent different environments with different distinct exponents. Hewett (1986) supplies the first distinct evidence that the porosity logs are perpendicular to the bedding may follow the statistics of fGn, while those parallel with the bedding follow fBm. The fBm and fGn analysis of the porosity log was considered by Hardy and Beier (1999), and recently by Sahimi (2011). One important outcome of the well logging data following the statistics of fBm or fGn is the existence of the long-term correlations. More recent research shows that densities, seismic velocities and elastic module of rocks may also follow self-affine distributions, like the fBm and fGn

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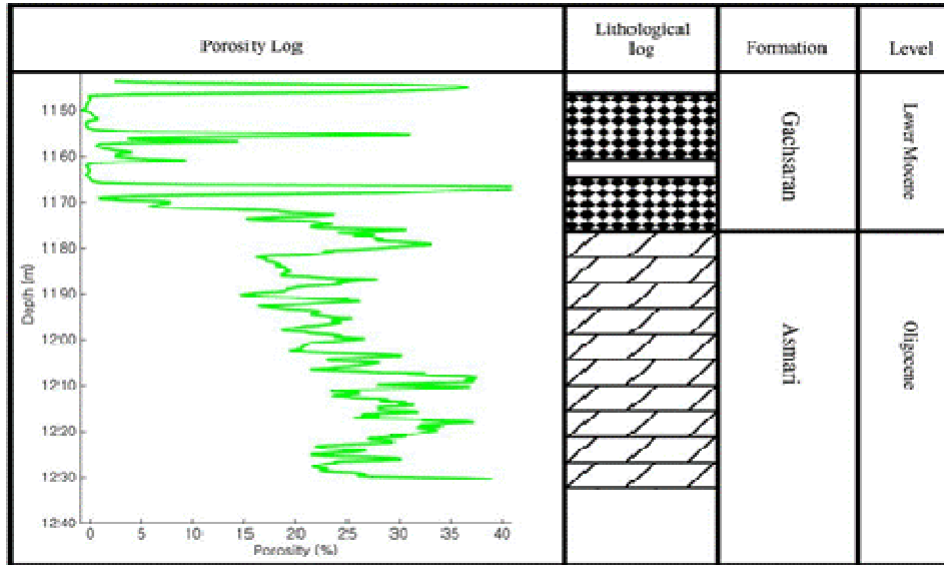


Figure 1. Geological time scale and neutron porosity well log of Farour-A1.

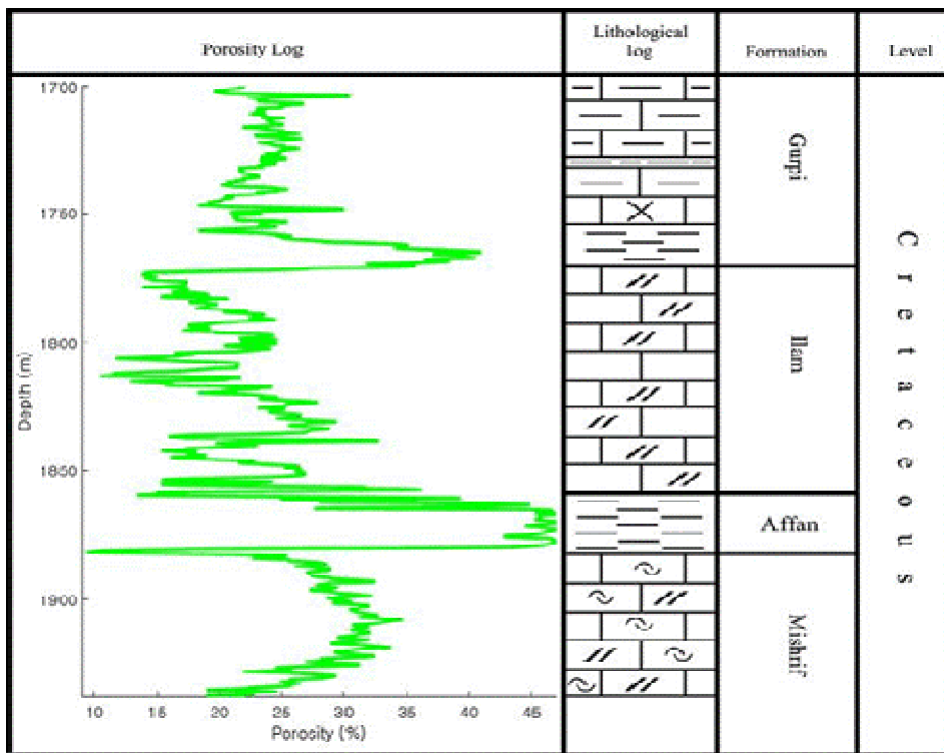


Fig. 2. Geological time scale and neutron porosity well log of Farour-B2.

(Sahimi and Tajer, 2005).

### Farour-A and Farour-B Offshore Oil Filed

Well logging data used in this study, are from Farour-A and Farour-B offshore oil field located in Persian Gulf in

southern part of Iran. Depth resolution or sampling intervals of data are 20 centimeters. The geological column and neutron porosity log for the Farour-A1 and Farour-B2 wells are respectively shown in Fig.1 and Fig. 2. Well logging data examined in this research are four well logging series including bulk density (RHOB), sonic transmit time logs (DT), acoustic impedance (AI)

and neutron porosity (NPHI). The bulk density represents the overall density of a rock, including its solid matrix and fluid content. The sonic transmit time log measures acoustic velocity of rock, and it is related to the porosity and the fluid content of the pores. Acoustic impedance is the product of density and velocity value of the rock, and it has an inverse relationship to porosity. Neutron porosity measurements use a neutron source and account fluid content (hydrogen index) in the porous media and determine the porosity.

## METHODOLOGY

The research focuses on multifractality nature of well logging data. Thus, by analyzing the multifractality in the original, shuffled and surrogated well logging data, type of multifractality nature in the well logging data are distinguished.

### Monofractal and multifractal time series

Multifractal analysis is currently considered as a useful and major tool for understanding the structure in a set of data (Karacan, 2009). The analysis consists of measuring the scaling exponents of the data. The exponents may be used for detection, classification and interpretation of the data. The scaling property for all scales  $a > 0$  is:

$$B(at) \equiv a^H B(t) \quad (1)$$

where  $\equiv$  is an equality of finite dimensional distribution. A covariance analysis shows that the fBm are a nonstationary process, and its increment is stationary so it can define a generalized power spectrum with a power law decay by an exponent  $\beta = 2H + 1$  and  $1 < \beta < 3$  (Turcotte, 1992). Thus, any individual realization of the process is a fractal curve with a fractal dimension  $D$  calculated by:  $D = 2 - H$ . fBm realization is everywhere singular, i.e.,  $tR$  and for  $0 < a < 1$ , the following relationship is proven:

$$|B_H(t) - B_H(t-t)| \leq K |t|^\alpha \quad (2)$$

where  $K$  is a positive constant.

The Lipschitz regularity of a time series at point  $t_0$  is the domain in which above equation is verified. If it is  $a < 1$  at point  $t_0$ , the signal is not differentiable and will characterize the singularity type. The Holder exponent of fBm is equal to the Hurst parameter where the following relationship is satisfying (MALLAT, 1998).

$$|B_H(t) - B_H(t-t)| \sim |t|^H \quad (3)$$

Multifractality can be studied with two ways of the local and the global nature. The local nature is a local estimation of Lipschitz regularity, and the global nature consists of defining a method to estimate the global

repartition of the various Holder exponents. Global multiscale analysis will help to show the data is monofractal or multifractal.

Holder exponent  $H > 0.5$  represent positive correlation and persistent time series and  $H < 0.5$  represent negative correlations and anti-persistence time series, while  $H = 0.5$  implies that successive increments of the data are random and follow a Brownian motion.

Typically, broad probability density functions of the data with a fat tail and long-term correlations are the two factors that cause multifractality in stochastic series (Movahed et al., 2006, Jafari et al., 2007, Zunino et al., 2009). By investigating the multifractality in original, shuffle and surrogate time series, type of multifractality is distinguished. It is possible to find the nature of multifractality caused by: (I) Non-Gaussian PDFs of the data and their long tails, or (II) Different long-term correlations of the small and large fluctuations, or (III) Both. In case (I) multifractality cannot be destroyed by shuffling the series. Moreover,  $h(q)$  of the surrogate series will be independent of  $q$ . In case (II) multifractality is removed by shuffling because it destroys all the correlations. Therefore, if multifractality is only due to the long-term correlations, in the shuffled series,  $h(q=2) = 0.5$ , i.e. the shuffled series is monofractal. If

multifractality is the results of both factors, then both the shuffled and surrogate series will show weaker multifractality than the original series.

### Shuffle and surrogate data

Two procedures are needed for analyzing the nature of the multifractality in time series, shuffling and surrogating. Shuffling randomizes the order of the time series value, all the spatial correlations are destroyed but the probability density function (PDF) of it will not be affected by the shuffling.

Using surrogate data in the nonlinear time series analysis was introduced by Theiler et al. (1992). Surrogate series are generated from the original time series for determining the effect of the broadness of PDF (non-Gaussian PDF). The measured topological properties of the original time series are then compared with the measured topological properties of the surrogate time series. If both original time series and the surrogate time series yield the same values for the topological properties, the null hypothesis that the data set is random noise, cannot be ruled out. A distinct algorithm for generating this, is as follows (Mazaraki, 1997):

1. Input the experimental time series data  $x(t_j), j = 1, \dots, N$  into a complex array.
$$z(n) = x(n) + iy(n), n = 1, \dots, N \quad (4)$$
 where  $x(n) = x(t_n)$  and  $y(n) = 0$ .
2. Construct the discrete Fourier transform.

$$Z(m) = X(m) + iY(m) = \frac{1}{\sqrt{N}} \sum_{n=1}^N x(n) e^{-2\pi i(m-1)(n-1)/N} \quad (5)$$

3. Construct a set of random phases.

$$\varphi_m \in [0, \pi], \quad m = 2, 3, \dots, \frac{N}{2} \quad (6)$$

4. Apply the randomized phases to the Fourier transformed data.

$$Z(m) \quad \text{for } m=1 \text{ and } m = \frac{N}{2} + 1$$

$$z(m) = \begin{cases} Z(m) & \text{for } m=1 \text{ and } m = \frac{N}{2} + 1 \\ \left| Z(m) \right| e^{i\varphi_m} & \text{for } m=2, 3, \dots, \frac{N}{2} \end{cases} \quad (7)$$

5. Construct the inverse Fourier transform of  $Z(m)'$ .

$$z(n)' = x(n)' + iy(n)' = \frac{1}{\sqrt{N}} \sum_{m=1}^N z(m)' e^{2\pi i(m-1)(n-1)/N} \quad (8)$$

### Multifractal Detrended Fluctuation Analysis (MF-DFA)

MF-DFA was proposed by Castro e Silva and Moreira (1997). After that, It was modified by Kantelhardt et al. (2002) and represents a generalization of the DFA approach that has been used for the analysis of various types of stochastic series in science and engineering, such as climate change, and finance and economic activities, heart rate dynamics, DNA sequences, human gait and neuron spiking and seismic data (Telesca et al., 2004, Shadkhoo and Jafari, 2009, Manshour et al., 2009; 2010) and so forth. The multifractal DFA (MF-DFA) procedure consists of five steps (Kantelhardt et al., 2002). Three first steps are essentially identical to the conventional DFA procedure. Let us assume that  $(x_i)$  is a series of length  $N$ , the steps are as follows:

Step1: determining the profile  $Y(j)$  by integrating the time series.

$$y(k) = \sum_{i=1}^k \frac{x(i) - \bar{x}}{N} \quad k = 1, \dots, N \quad (9)$$

where  $\bar{x}$  represents the average value.

Step 2: dividing the profile  $Y(j)$  into  $N_s = \text{int}(N/s)$  non-overlapping segments of equal length  $s$ . for contributing all the data when  $N$  is not a multiple of  $s$ , the same procedure is repeated starting from the opposite end. So,  $2N_s$  segments are obtained.

Step 3: Calculating the local trend for any  $2N_s$  segments by  $m$  degree least square fit of the profile and then determining the variance for each segment.

$$Y_s(j) = Y(j) - y_{v,s}^m(j) \quad (10)$$

$$F_{DFA}^2 m(v, s) = \frac{1}{s} \sum_{j=1}^s Y_s^2(j) \quad (11)$$

Step 4: average over all segments to obtain the  $q$ th order fluctuation function.

$$F_q(s) = \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F_{DFA}^2 m(v, s)]^{1/q} \approx s^{h(q)} \quad (12)$$

for  $q = 0$

$$F_0(s) = \exp \left[ \frac{1}{2N_s} \sum_{v=1}^{2N_s} \ln [F_{DFA}^2 m(v, s)] \right] \approx s^{h(0)} \quad (13)$$

Step 5: Determine the scaling behavior of the fluctuation function by analyzing log-log plot of  $F_q(s)$  versus  $s$  for each value of  $q$ .

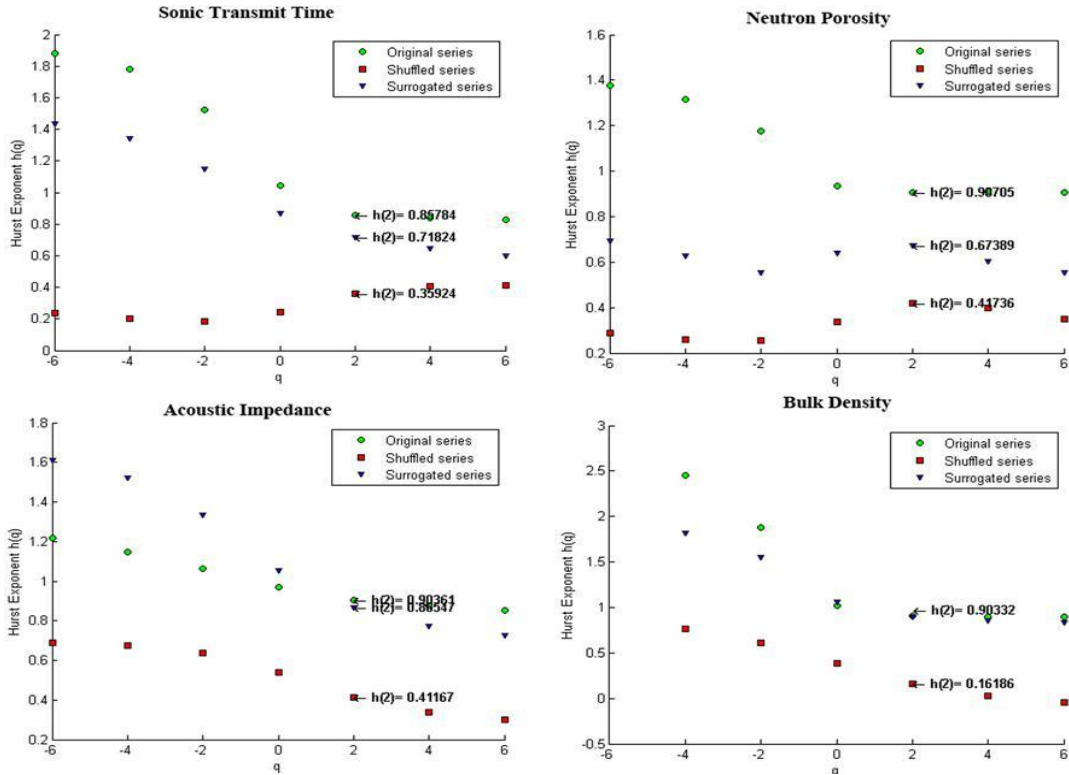
For very large scales,  $s > N/4$ ,  $F_q(s)$  becomes statistically unreliable because the number of segments  $N_s$  for the averaging procedure in step 4 becomes very small So, scales  $s > N/4$  should be excluded from the fitting procedure for determining  $h(q)$ . Also, systematic deviations from the scaling behavior occur for very small scales,  $s = 20 - 30$  and so we ignore small scales too. The Hurst exponent  $H$  is given by,  $H = h(q = 2)$ , where  $h(q)$  is the generalized Hurst exponent obtained from step 5. If  $h(q)$  varies with  $q$ , the series shows multifractality, whereas a constant  $h(q)$  is a characteristic of monofractal system.

For the positive value of  $q$ ,  $h(q)$  describes the scaling behavior of the segment with large fluctuations and for the negative values of  $q$ ,  $h(q)$  describes the scaling behavior of the segments with small fluctuations. Usually the large fluctuations are characterized by a smaller scaling exponent  $h(q)$  for multifractal series than the small fluctuations. For stationary time series, the exponent  $h(2)$  is identical to the Hurst exponent.

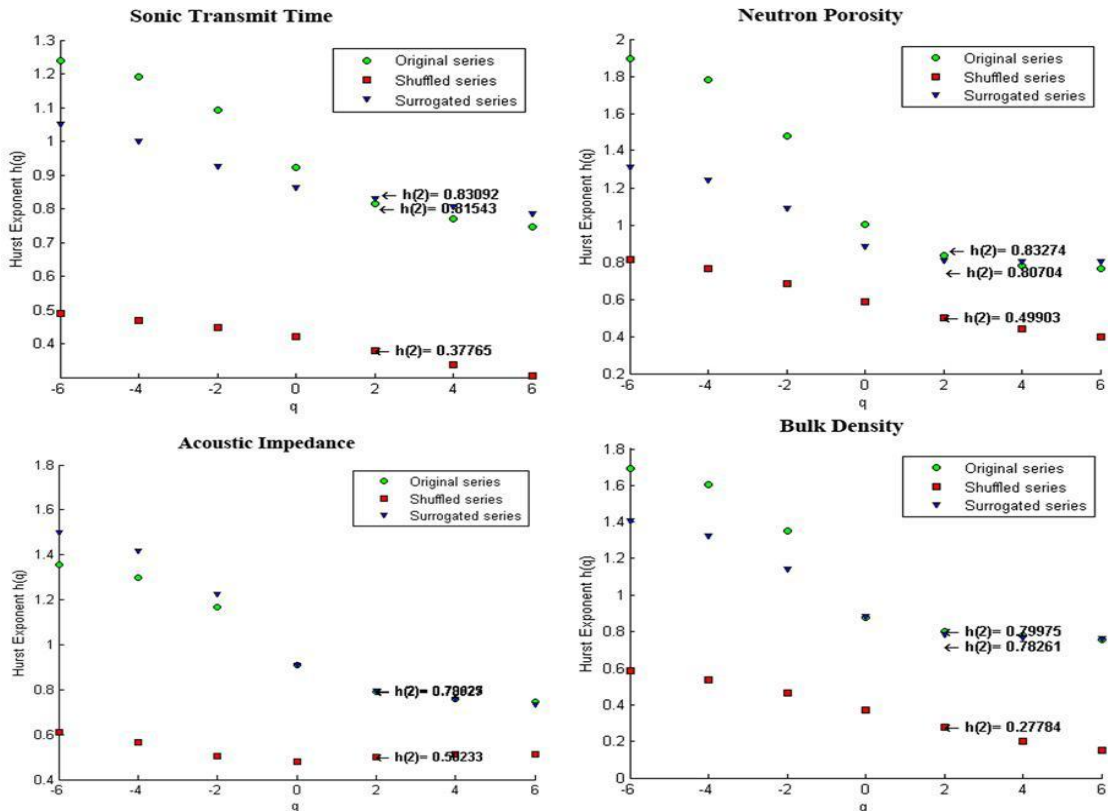
### RESULTS AND DISCUSSION

MF-DFA that was performed on well logs from the well Farour-A1 belongs to Farour-A offshore oilfield and the well Farour-B2 belongs to Farour-B offshore oilfield.

Hurst exponents  $h(q = 2)$  of Farour-A1 and Farour-B2 well logs are shown on Fig. 3 and Fig. 4. Hurst exponents for all well logs from these two wells are greater than 0.5 and represent positive correlation and persistent time series. These results disaccord to Ferreira et al (2009) on Namorado sandstone field (an offshore



**Fig. 3.** Generalized Hurst exponent for neutron porosity, acoustic impedance, bulk density and sonic transmit time logs in Farour-A1.



**Fig. 4.** Generalized Hurst exponent for neutron porosity, acoustic impedance, bulk density and sonic transmit time logs in Farour-B2.

Brazilian oil field) with negative correlation. This behavior is due to the different depositional environment. While Namorado sandstone field is formed by the coalescence of channels and lobes deposits on the irregular depositional surface with elongate, dome shape, partially faulted structure due to salt flow activity in the late Cretaceous era (Bacoccoli, 1980) and Farour fields is formed by interbedded carbonates, sandstone and shale with a poor matrix porosity and permeability. In spite of poor primary reservoir maturity, these reservoirs have the extensive production rate due to tectonic activity (Hull, 1970, Bacoccoli, 1980).

Generalized Hurst exponent  $h(q)$  for Farour-A1 and Farour-B2 well logs in original, shuffled and surrogated forms are respectively shown in Fig. 3 and Fig. 4. Generalized Hurst exponents of the surrogated series have no monofractal behavior and are not independent on  $q$ . So, multifractality nature of the series is not just due to Non-Gaussian PDFs of the data and their long tails (type I). Generalized Hurst exponents of the shuffled series have no monofractal behavior and are not independent on  $q$ . So, the correlation is not destroyed and multifractality nature of the series is not just due to the different long-term correlations of the small and large fluctuations (type II). Generalized Hurst exponents of the shuffled and surrogated series have weaker multifractality than the original series. So, multifractality is caused by both factors of Non-Gaussian PDFs and different long-term correlations of the small and large fluctuations (type III). This behavior is related to physical properties of the earth in which, depositional environment over the time scale increases the long-term correlation and strong heterogeneity causes broad probability distribution. So, different layers have different power law exponent.

## CONCLUSION

Multifractal Detrended Fluctuation Analysis (MF-DFA) is a good procedure to study multifractality nature of the well logging data by considering original, shuffled and surrogated series. MF-DFA examined on four types of well logging data belong to two different wells with different geological time scale. These analyses showed that Hurst exponents were greater than 0.5 and exhibited persistence in the long-term correlation of these data. And also, multifractality nature of them was due to the long-term correlation and broad probability distribution due to physical properties of the ground in which, depositional environment over the time scale rises in long-term correlation and strong heterogeneity gives broad probability distribution.

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