

*Full Length Research Paper*

# The evaluation of the full-factorial attraction model performance in brand market share estimation

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Having reliable models to predict the influence of marketing instruments on market performance is critical for efficient allocation of marketing resources. An Attraction Model is often used to predict market share, although it does not account for changing market structure. Howie and Kleczyk (2007) proposed a joined pre- and post- new brand introduction model based on a re-conceptualization of market share as a series of two-brand groups. The model is evaluated on its reliability of market share estimation. The study results reveal that the Full-Factorial Attraction Model accounts for the changing market structure as well as has a high predictive power.

**Key words:** market share estimation, Full-Factorial Attraction Model, GLS estimation.

## INTRODUCTION

Having reliable models predicting the influence of marketing instruments on actual market performance is critical to ensuring that marketing resources are allocated efficiently. The leading approach to understanding and modeling market performance is based on the concept that market share is a function of the product attractive-ness share (Cooper and Nakanishi, 1996). This concept has been formalized in the "Attraction Model." The Attraction Model is, however, limited in its ability to predict market share due to severe practical data limitations, including changing market structures across product classes (Cooper and Nakanishi, 1996).

To resolve the problem, Howie and Kleczyk (2007) proposed a Full-Factorial Attraction Model that accounts for the changing market structure across products as it re-conceptualizes any market share as a series of two-brand groups. Due to the transformation, while the number of 2-product groups changes with a competitor's entry/exit, the structure of each group remains constant despite a change in the number of competitors (Howie and Kleczyk, 2007). In their 2008 article, Howie and Kleczyk compared the parameter estimates of the Attraction and Full-Factorial Attraction Model. They found the Full-Factorial Attraction Model to be a more reliable specification as it controls for structural changes occurring

from a new brand entrance to the market (Howie and Kleczyk, 2008).

The objective of this paper is to extend the article by Howie and Kleczyk (2008) and explore the performance of the Full-Factorial Attraction Model over the Attraction Model in the market share predictions. Based on the results of Howie and Kleczyk's (2008) study, a model specification accounting for the impact of the marketing instruments, as well as the entry of a new competitor to the market is employed for the comparative analysis. The study evaluates the estimation results as well as the performance of each model based on the model fit compared to the actual historical data.

## THEORETICAL BACKGROUND

### The attraction model

Market share is usually defined as the percentage of the total available market serviced by a brand (Giovan et al., 1997). This definition is translated to the following scientific description of the market share concept. The market share of brand  $i$  at time  $t$  ( $M_{it}$ ) is equal to its attraction relative to the sum of all attractions (Cooper and Nakanishi, 1996):

$$M_{it} = A_{it} / \sum_{j=1}^I A_{jt} \quad \text{for } i = 1 \dots I.$$

$A_{it}$  is the attraction of brand  $i$  at time  $t$ , given by:

$$A_{it} = \exp(\alpha_i + \beta_i t) \prod_{j=1}^I \prod_{k=1}^K f(x_{kjt})^{k_{jt}} \quad \text{for } i = 1 \dots I \text{ and } t = 1 \dots T$$

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Where  $x_{kjt}$  is the  $k$ -th explanatory variable for brand  $j$  at time  $t$ ,  $f(x_{kjt})$  denotes a transformation function of the  $k$ -th explanatory variable ( $x_{kjt}$ ) for brand  $j$  at time  $t$  and where  $k_{jt}$  is the corresponding coefficient for brand  $i$ . The parameter  $i$  is a brand-specific constant and the error term  $(\epsilon_{it} \dots \epsilon_{jt})'$  is normally distributed with zero mean and as a non-diagonal covariance matrix (Cooper and Nakanishi, 1996).

The Attraction Model employs the concept of market share with the unobserved  $A_{it}$ . There are two types of Attraction Model specifications: Multiplicative Competitive Interaction (MCI) and Multinomial Logit (MNL) (Fok, 2003):

$$\text{MCI: } A_{it} = \exp(i + it) \prod_{j=1}^I \prod_{k=1}^K k_{jt} x_{kjt} \text{ for } i = 1 \dots I \text{ and } t = 1 \dots T$$

$$\text{MNL: } A_{it} = \exp(i + it + \prod_{j=1}^I \prod_{k=1}^K (k_{jt} x_{kjt})) \text{ for } i = 1 \dots I \text{ and } t = 1 \dots T.$$

The Attraction Model can be transformed utilizing a brand-based transformation approach resulting in the following Attraction Model definition:

$$\ln M_{it} - \ln M_{jt} = (i - j) + \prod_{k=1}^K (k_{ji} - k_{jl}) \ln x_{kjt} + \prod_{p=1}^P ((p_{ji} - p_{jl}) \ln M_{j,t-p} + \prod_{k=1}^K (p_{kji} - p_{kjl}) \ln x_{k,t-p}) + \epsilon_{it} \text{ for } i = 1 \dots I-1$$

Where  $M_{it}$  is the market share for brand  $i$  at time  $t$  and the error variable is defined as  $\epsilon_{it} = \epsilon_{it} - \epsilon_{jt}$  with  $(\epsilon_{1t} \dots \epsilon_{jt})'$  being normally distributed (Fok, 2003).

There are four restrictions often put on the Attraction Model in order to reduce its specification: (1) restricted covariance matrix ( $\text{diag}(\sigma_1^2, \dots, \sigma_{I-1}^2) + \sigma_1^2 \mathbf{1}_{I-1} \mathbf{1}'_{I-1}$ ); (2) restricted competition ( $k_{ji} = 0$  for  $j \neq i$ ); (3) restricted effects ( $k_{ji} = 0$  for  $j \neq i$ ); and (4) restricted dynamics ( $p_{ji} = 0$  for  $j \neq i$ ). When the Attraction Model is constrained using these four restrictions listed above, the following model specification is employed (Fok, 2003):

$$\ln M_{it} - \ln M_{jt} = (i - j) + \prod_{k=1}^K (k_{ji} - k_{jl}) \ln x_{kjt} + \epsilon_{it}, \text{ for } i = 1 \dots I-1.$$

### The full-factorial attraction model

The Full-Factorial Attraction Model is based on a re-conceptualization of any market share variable for each brand as a series of two-product markets (Howie and Kleczyk, 2007). The transformed two-product market is described as follows:

$$m_{ijt} = M_{it}/(M_{it} + M_{jt}), \text{ where } i = 1 \dots I-1 \text{ and } j = 1 \dots I-1 \text{ and } i \neq j; t = 1 \dots T$$

$$m_{jit} = M_{jt}/(M_{it} + M_{jt}), \text{ where } i = 1 \dots I-1 \text{ and } j = 1 \dots I-1 \text{ and } i \neq j; t = 1 \dots T$$

$$m_{ijt} + m_{jit} = 1, \text{ where } i = 1 \dots I-1 \text{ and } j = 1 \dots I-1 \text{ and } i \neq j; t = 1 \dots T$$

for  $I(I-1)/2$ , where  $i, j$  are brands in a market. As a result, at any point in time,  $t$ , the average market share is equal to

50% (Howie and Kleczyk, 2008). The independent variables are transformed to conform to the new way of data definition. A difference between two products' independent variables is computed (Howie and Kleczyk, 2008):

$$xgap_{ijt} = x_{it} - x_{jt}, \text{ where } i = 1 \dots I, j = 1 \dots I \text{ and } i \neq j; t = 1 \dots T.$$

Based on the above variable specification, the Full-Factorial Attraction Model is defined as follows (Howie and Kleczyk, 2007):

$$m_{ijt} = i + xgap_{ijt} + \epsilon_{it}, \text{ where } i = 1 \dots I \text{ and } j = 1 \dots I \text{ and } i \neq j; t = 1 \dots T.$$

It is important to note a caveat to the Full-Factorial Attraction Model with regards to the model estimation. Although the regression estimates derived based on this model are unbiased, the coefficients' standard errors are biased down due to the increased sample size from the data transformation (Howie and Kleczyk, 2007). To correct for this estimation problem, the standard error values are adjusted by a factor of the square root of 2 (Kohler and Rodgers, 2001).

### Data

The data set utilized in this study is the same data set as utilized by Howie and Kleczyk in their 2008 article and includes three hypertension products from April 2002 to February 2006. While two of the products existed continuously during this time span, the third product entered the market in February 2003. The dependent variable is the actual market share based on the number of prescriptions filled at retail pharmacies for each product. Since a large portion of prescriptions actually filled at a pharmacy are for refills, where the brand decision had been made previously and, thus, not influenced by marketing efforts, prescription volume is defined in terms of new prescriptions. The marketing instrument evaluated in this study is "detail" share of voice. Despite the increasing use of the Direct-to-Consumer advertising, for many pharmaceutical brands, the largest promotional spend by pharmaceutical companies remains for a personal field force. The field force typically consists of hundreds of sales representatives calling on physicians and communicating the value of their product through a face-to-face interaction. This face-to-face interaction is called a "detail". Market share is defined as the products' prescribing volume expressed as a percentage of all prescriptions in the defined market. Share-of-voice is defined as the products' "detail" volume expressed as a percentage of all "details" in the defined market (Note: The data set is proprietary to TargetRx, Inc).

**Model specification:** In order to analyze the hypertension market share data, the performance of both the Attraction and Full-Factorial Attraction Model is evaluated. The restricted covariance, competition, effects and

dynamics are assumed for this investigation as suggested by Fok (2003). Following Howie and Kleczyk (2008), the Attraction and Full-Factorial Attraction Model specifications testing for the structural break, from the effect of marketing instruments and a market structure, are selected for market share estimation.

### The attraction model

To transform market share data for the Attraction Model estimation, the brand-based approach is utilized. Following Fok et al. (2001) and Howie and Kleczyk (2008) methodology of testing for a structural break in the attraction of brand  $j$ , the variable  $exp(D_t^*)$  is included in the specification of brand  $j$ .  $D_t^*$  represents a dummy variable that takes a value of 1 for three-brand market size and 0 otherwise. The structural break can also be a result of the effect of marketing instruments on market share. To test for this effect, the independent variable is interacted with a market structure dummy variable ( $exp(D_t^*) \ln x_{kjt}$ ). If the dummy variable and the interaction variable are found statistically significant, then the estimated model specification does not account for varying market structure and the change in impact of marketing variables on market share. Based on the above specifications, the model is presented as follows (Fok, 2003):

$$\ln M_{it} - \ln M_{it-1} = (\alpha_i - 1) + \sum_{j=1}^I \sum_{k=1}^K (\alpha_{kji} - \alpha_{kji-1}) \ln x_{kjt} + \beta_i \exp(D_t^*) \ln x_{kjt} + \beta_i \exp(D_t^*) + \epsilon_{it},$$

for  $i = 1 \dots I-1, t = 1, \dots, T$ .

In order to compute and compare the market share prediction to the historical series, the estimated Attraction Model coefficients are multiplied by the logarithmically transformed independent variables and the following system of equation is solved:

$$\sum_{j=1}^I M_{it} = 1$$

$$M_{it} / M_{it-1} = \exp(\beta_i \ln x_{kjt} + \beta_i \exp(D_t^*) \ln x_{kjt} + \beta_i \exp(D_t^*))$$

for all  $i = 1 \dots I-1$ ,

Where  $\beta$  identifies the estimated coefficients. These market share series are compared with the actual historical brand performance (Fok, 2003).

### The full-factorial attraction model

As suggested by Fok et al. (2001) and Howie and Kleczyk (2008), the Full-Factorial Attraction Model specification testing for the structural break, from the effect of marketing instruments and a market structure, is selected for market share estimation. As a result, the model is specified as follows (Howie and Kleczyk, 2008):

$$\ln m_{ijt} = \alpha_i + x_{gapijt} + \beta_i \exp(D_t^*) x_{gapijt} + \beta_i \exp(D_t^*) + \epsilon_{it}$$

Where  $i = 1 \dots I$  and  $j = 1 \dots I$  and  $i, j; t = 1 \dots T$ ,

Where  $exp(D_t^*)$  denotes market structure dummy variable that takes a value of 1 for a three-brand market size and 0 otherwise,  $exp(D_t^*) x_{gapijt}$  represents the interaction variable between the share of voice variable and the market structure dummy. If the dummy variable and the interaction variable are found statistically significant, then the estimated model specification does not account for varying market structure and the change in the impact of marketing variables on market share.

The Multinomial Logit Model specification is utilized in this model to transform the data. The transformation was not employed in Howie and Kleczyk's (2008) article as their objective was not to test for the model's performance. The Multinomial Logit Model specification allows for only the left-hand side of the regression to be logarithmically transformed. The benefit of this specification is the normalization of the dependent variable distribution and therefore enforcement of the market share estimate between 0 and 1 (Cooper and Nakanishi, 1996).

To obtain the estimate of market share of brand  $i$  at time  $t$ , utilizing the Full-Factorial Attraction Model, require "adding up" market shares of each brand combination times the relative shares of the original share of each brand pair. The parameter coefficients of the independent variables are utilized to estimate the market share of each brand combination. The following points outline the necessary steps for this conversion (Howie and Kleczyk, 2007):

**Step 1:** Put all predicted brand market shares in terms of  $pM_{it} \dots pM_{It}$ .  $M_{it} \dots M_{It}$  are the actual values of market share from the last month with observed data.

$$pM_{ijt} = pM_{it} / (M_{it} + M_{jt}), \text{ where } i = 1 \dots I, j = 1 \dots I \text{ and } i, j; t = 1 \dots T$$

$$pM_{ijt}^* (M_{it} + M_{jt}) = pM_{it}, \text{ where } i = 1 \dots I, j = 1 \dots I \text{ and } i, j; t = 1 \dots T.$$

**Step 2:** Take the average value of market share for each brand ( $pM_{1t} \dots pM_{It}$ ) at time  $t$ .

$$pM_{1t} = pM_{1t} / (I-1), \text{ where } i = 1 \dots I; t = 1 \dots T$$

$$pM_{It} = pM_{It} / (I-1), \text{ where } i = 1 \dots I; t = 1 \dots T.$$

**Step 3:** Rebalance  $pM_{1t} \dots pM_{It}$  in order for the brand's market share to sum up to unity:

$$pM_{1t} + \dots + pM_{It} = 1, \text{ where } i = 1 \dots I; t = 1 \dots T.$$

### Estimation procedures

The Attraction and Full-Factorial Attraction model specifications are estimated utilizing the GLS-SUR estimation method. For the regression model with restricted covariance, competition, restricted effects, and dynamics, the iterative GLS-SUR estimator is (Fok, 2003):

$$\beta^{hat}_{SUR} = (X' (X^{hat-1} I_T X)^{-1} X' (X^{hat-1} I_T) y).$$

As the covariance matrix ( $\hat{\Sigma}^{hat}$ ) is unknown in the GLS estimation, a new estimate ( $\hat{\Sigma}^{bar}$ ) is computed, where  $\hat{\Sigma}^{hat}_t$

**Table 1.** Attraction model with share of voice and market structure dummy variable interaction (OLS Estimation).

Variable	Coefficient	Std. Error	Std. Coefficient	t-Statistic	Prob.
Intercept	-1.061	0.051		-20.87	0.000
Share of Voice Gap	0.237	0.034	0.073	6.877127	0.000
Market Structure Dummy	-0.609	0.307	-0.294	-1.984	0.054
Share of Voice Gap* Market Structure Dummy	1.557	0.344	0.672	4.523	0.000

Note: R-square = 0.996; Adjusted R-square = 0.995; F-statistic = 3601.557; Prob (F-statistic) = 0.000  
Source: Howie and Kleczyk (2008).

**Table 2.** Full-factorial attraction model with share of voice and market structure dummy variable interaction (GLS Estimation).

Variable	Coefficient	Std. Error	Std. Coefficient	t-Statistic	Prob.
Intercept	-0.881	0.031		-28.058	0.000
Share of Voice Gap	3.863	0.155	0.540	24.960	0.000
Market Structure Dummy	-0.074	0.047	0.002	-1.594	0.113
Share of Voice Gap* Market Structure Dummy	4.742	0.552	0.428	8.588	0.000

Note: R-square = 0.759; Adjusted R-square = 0.756; F-statistic = 169.928; Prob (F-statistic) = 0.000

## RESULTS

Following Howie and Kleczyk (2008), the impact of the structural market change variable as well as the marketing variable on market share was estimated (i.e., share of voice). The Full-Factorial Attraction Model employed the Multinomial Logit Model market share transformation. The models were assumed to have restricted covariance, competitiveness, effects, and dynamics and were estimated utilizing the GLS-SUR methodology (Fok, 2003). Note that the Attraction Model regression results were replicated based on Howie and Kleczyk's (2008) specification for the purpose of this study. For detailed description of the results, please refer to their 2008 article.

### The attraction model

As found by Howie and Kleczyk (2008), the share of voice variable for the two-product market had a parameter of 0.237 while a three-product market had a coefficient of -0.454 (Table 1). The result indicates the marketing variable estimate being sensitive to the number of brands in the market. Additionally, the standardized share of voice coefficient has a lower value than the standardized coefficient on the dummy variable. This finding implies the market structure variable as having a greater impact on market share compared to the marketing variables. As in Howie and Kleczyk (2008), the OLS estimation was employed instead of GLS, due to a nearly singular matrix.

### The full-factorial attraction model

In the case of the Full-Factorial Attraction Model, the separate two- and three-brand share of voice impact on the market share was 3.863 for two-brand market and 8.575 for three-product market (Table 2). The standardized coefficients were also different for the two- and three- brand markets (0.540 vs. 0.428). The standardized coefficient on the market structure dummy variable was only 0.002, implying that the market share is mostly explained by the marketing variable.

### Model comparisons

As found in Howie and Kleczyk (2008), the Attraction Model does not account for the changing market structure from a new brand entry to the market. The market structure dummy variable is also statistically significant and carries a large positive coefficient value. On the other hand, the Full-Factorial Attraction Model captures the changing market structure as the market structure dummy variable is statistically insignificant. As found by Howie and Kleczyk (2008), the marketing parameters are responsive to market changes in both models. Another approach to determining the relative performance of the two models is to use each specification to predict actual performance. For ease of presentation, the monthly total market volume is the actual market volume and the share predictions are converted to overall prescription volume (NRx). As there are three products present in the analysis noted as Base Prod, Prod 1, and Prod 2 (new brand),

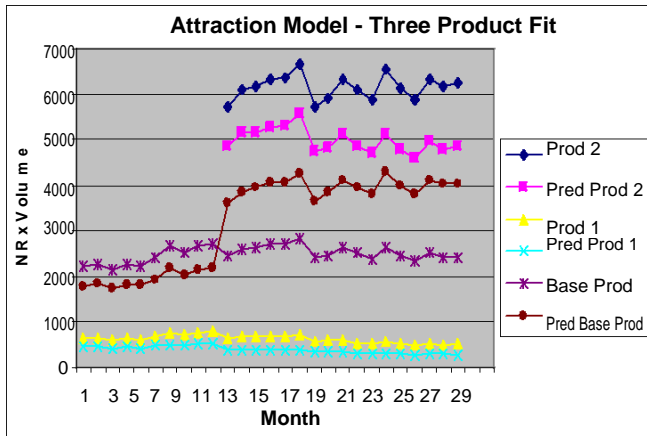


Figure 1. Attraction model – Three-Brand fit.

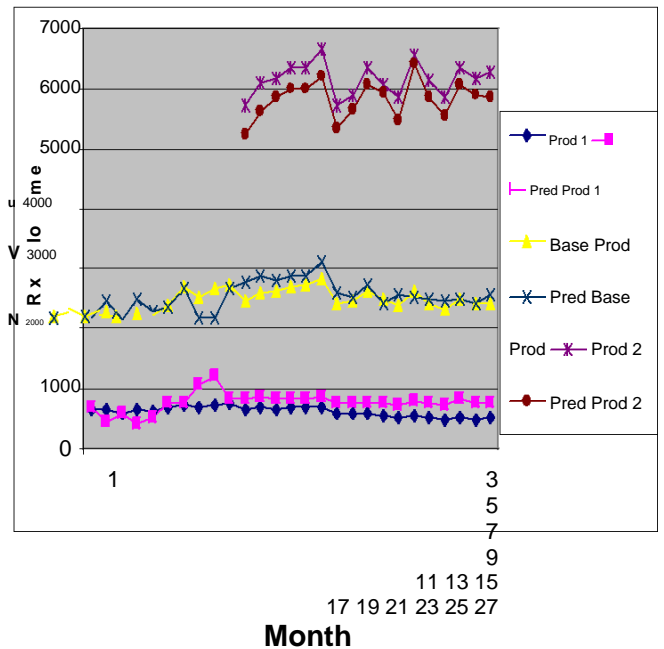


Figure 4. Full-Factorial attraction model – New entrant predictions.

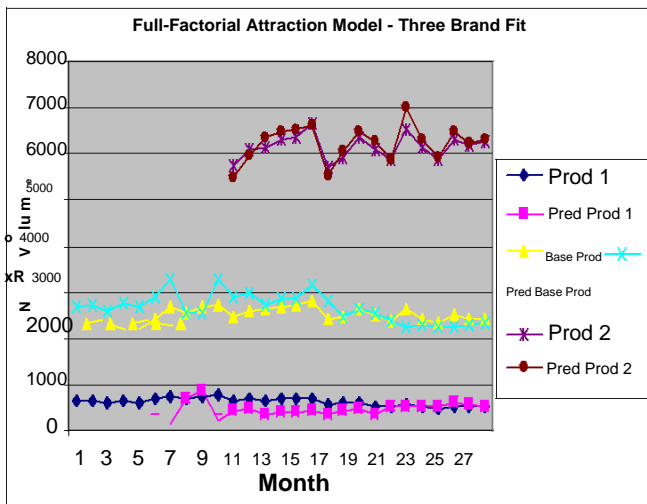


Figure 2. Full-Factorial attraction model – Three-Brand fit.

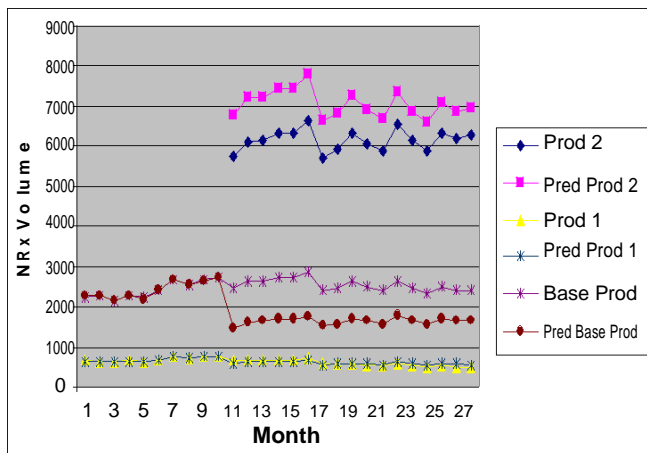


Figure 3. Attraction model – New entrant predictions.

their historical NRx series as well as predicted NRx series (notation: Pred) are compared.

The results from this initial test indicate that the Full-Factorial Attraction Model performs better than the Attraction Model in fitting the actual data series. As shown on Figure 1, the Attraction Model does not account for the structural shift in NRx volume due the third brand's (Prod 2) entrance to the market. In particular, the shift in the Pred Base Prod NRx series is pronounced. For the Full-Factorial Attraction Model, the structural change of the model fit is due to the third brand (Prod 2) introduction is not observed (Figure 2). All historical and predicted NRx series present no structural break. By comparison, the

Full-Factorial Attraction Model sum of squared errors is 15% of the sum of squared errors for the Attraction Model (6,517,212 compared to 44,639,187) which implies lower variability of predicted NRx series.

The second test of the two models compares actual predicted performance when the model is built using the data prior to the introduction of the third competitor. With new competitor's frequent market entries/exits, it is critical to know whether parameter estimates, from historical time-periods, can be confidently used when the market structure changes. Again, the Full Factorial Attraction Model outperforms the Attraction Model considerably as none of the predicted NRx product series experiences a structural break (Figures 4). On the other hand, Base Prod presents a shift in the predicted NRx series (Figure 3) implying lack of accounting for the structural change in the model specification. By comparison, the Full-Factorial Attraction Model sum of squared errors is 23% of the sum of squared errors for the Attraction Model (6,825,813 compared to 29,587,990), which again implies lower volatility in NRx series.

## Conclusions

This study investigated the performance of the Full-Factorial Attraction Model over the Attraction Model in obtaining reliable parameter estimates of the marketing variables when the market structure changes due to the exit/entry of a competitor.

In order to evaluate which model specification performs “better” in market share prediction, each model was estimated and tested against actual prescription data. In agreement with Howie and Kleczyk’s (2008) article, the regression results imply lack of accounting for market structure changes by the Attraction Model; while they also imply a full capture of the phenomenon by the Full-Factorial Attraction Model. Furthermore, a test of the predictive power based on a model fit compared to actual history, showed that the Full-Factorial Model outperformed the Attraction Model. A second test of the predictive power was performed using only the data available prior to the introduction of a new competitor. The forecasted series were compared to actual performance of the data after the introduction of the new competitor. The results of this test also favored the Full-Factorial Attraction Model.

The above analysis is the second empirical test of the Full-Factorial Attraction Model and provides support for this model as a better approach to modeling competitive markets. Having correct estimates of the responsiveness to marketing instruments is critical to proper resource allocation. While the focus of the article is on the entry/exit of a competitor, this approach can be extended to modeling any dynamic competitive situation.

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