

Full Length Research Paper

Computational design of cardiac activity

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Cardiovascular physics is an interdisciplinary research area where the ideas and methods from mathematics, physics, and biology are synergistically applied for advanced medical diagnosis. The research on cardiac activity includes the development of efficient quantitative techniques and theoretical approaches, as well as the use of mathematical reasoning to gain insight into the cardiovascular system. However, mathematical representation of the entire cardiac activity is still on the focus of the research of cardiovascular physics. Here in this paper, a simple computational model of cardiac activity has been designed through a cardiovascular system equation derived from the elementary fluid dynamics. The cardiovascular equation is expandable, by applying respective assumptions, therefore as the models of heart, blood flow rate, and blood pressure, the model of heart is flexible to represent the right and left ventricle, as well as right and left atria individually. The validity of the model has been estimated by deriving numerical figures of heart rate, end-diastolic volume, end-systolic volume, stroke volume, and ejection fraction. The analysis has also showed that the model can describe the abnormal conditions of the heart originated from heart blockage, leakage along with the normal operation. The validity of the models of blood flow rate and blood pressure has been verified by comparing the outcomes with some pre-established results in the physics of fluid dynamics.

Key words: Cardiovascular physics, cardiac activity, computational model, cardiovascular system equation, model of heart, blood flow rate, blood pressure.

INTRODUCTION

Computational design of any complex biological system is strongly dominated by mathematical modeling and computer simulation, which plays an important role in advancing our understanding of almost all areas of biology. A model is an abstraction of key components and processes of a system, and by constructing, simulating, and analyzing these simplified versions of the world, we are able to understand the mechanisms that link different components of the system, examine the consequences of these relationships, and predict how disturbances or modifications in one component or mechanism show up elsewhere. A crucial part of the modeling process is the evaluation of whether a proposed mathematical model can describe a system perfectly or not. The real world problem is firstly represented using mathematical equations, and then after applying assumptions, if required, and performing mathematical operations on the entire system, the model is developed.

Cardiovascular system is the blood distribution network in the body. The main components of the human

cardiovascular system are the heart, blood, and blood vessels. Heart, which consists of four chambers, is the mother component of the system, and is responsible for distributing the blood throughout the body via the blood vessels. A complete cycle of the cardiovascular system consists of two major parts: systole and diastole. The time required for a systole is approximately 0.35 s, while it is around 0.54 s for the diastole (Cromwell et al., 2004).

Many studies have been published in this field, and there are some important literatures on the functional imaging and modeling of the heart (Katila et al., 2001; Magnin et al., 2003). Some studies have been performed on the measurement of electrical activity, deformation, flows, fiber orientation (Masood et al., 2000; Rhode et al., 2005; MacLeod et al., 2001; Kilner et al., 2000; Faris et al., 2003), and on the modeling of the electrical and mechanical activity of the heart (Noble, 2004; Xia and Huo, 2003; Hunter et al., 2003; McCulloch et al., 1998).

This study provides a computational model of the cardiovascular system, in general, and the models of

heart, blood flow rate, and blood pressure, in particular. In whole literature, some assumptions have been considered, which include that although the heart's chambers are different in size, they are all considered being cylindrical shaped. They expand as the blood enters into it and contracts as the blood leaves it. The blood is considered to be the Newtonian fluid which is governed by the Navier-Stokes equation and by the continuity equation. Although the blood needs the help of the lungs for the supply of oxygen, its properties remain unchanged by the addition of that oxygen. Another assumption is required, and that is, the blood has both the radial and axial flow in only one direction - z-direction in a three

dimensional system. So, the other two components (x- and y-direction) are vanished.

MATERIALS AND METHODS

Developing the cardiovascular system equation

The velocity components in the x, y, and z directions are typically named u, v, and w respectively. Let, ρ be the density of blood, P the blood pressure, and μ is the kinematic viscosity of blood. Then neglecting the orientation of gravity inside the body, the Navier-Stokes equation in the Cartesian co-ordinates is given by the following equations (Acheson, 1990):

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (2)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (3)$$

With the assumption of no tangential velocity and no x and y components of velocity, a change of variables on the Cartesian equations will yield the following system of equations (Acheson, 1990):

$$\frac{\partial w}{\partial t} + f \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$$

$$\frac{\partial f}{\partial t} + f \frac{\partial f}{\partial r} + w \frac{\partial f}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} + \frac{f}{r^2} \right) \quad (5)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (rf) + \frac{\partial w}{\partial z} = 0 \quad (6)$$

Where f(r, z, t) be the radial flow component, w(r, z, t) be the axial flow component in z-direction. And the continuity equation reads:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho w)}{\partial z} = 0 \quad (7)$$

Now, define a new variable γ as $\gamma = r/R(z, t)$ where R(z, t) is the radius of the blood vessels, obviously the coordinate (r, z, t) is now replaced by coordinate (γ, z, t). Again, the velocity profile in the axial direction, w(η, z, t) is assumed to have the expression in the polynomial form (Yang et al., 1999):

$$w(\gamma, z, t) = \sum_{k=1}^N q_k (\gamma^{2k} - 1) \quad (8)$$

Where, q(z, t) is a another variable to be determined later. For simplification let, N=1. Then:

$$w(\gamma, z, t) = q(z, t) (\gamma^2 - 1) \quad (9)$$

And the velocity profile in the radial direction, w(η, z, t), is assumed to have the expression in the polynomial form (Yang et al., 1999):

$$f(\gamma, z, t) = \frac{\partial R}{\partial z} \gamma f + \frac{\partial R}{\partial t} \gamma - \frac{\partial R}{\partial t} \frac{\gamma}{N} \sum_{k=1}^N \frac{1}{k} (\gamma^{2k} - 1) \quad (10)$$

Again for simplification, let N=1, which implies (11):

$$f(\gamma, z, t) = \frac{\partial R}{\partial z} \gamma f + \frac{\partial R}{\partial t} \gamma - \frac{\partial R}{\partial t} \gamma (\gamma^2 - 1) \quad (11)$$

Using the help of equations of axial and radial velocity profile, (9) and (11) respectively, radial coordinate and the continuity equation (7), the Navier - Stokes equations get the forms as thus to determine the variable q(z, t) and R(z, t):

$$\frac{\partial q}{\partial t} - \frac{4q}{R} \frac{\partial R}{\partial t} - \frac{2q^2}{R} \frac{\partial R}{\partial z} + \frac{4\mu}{R^2} q + \frac{1}{\rho} \frac{\partial P}{\partial z} = 0 \quad (12)$$

$$2R \frac{\partial R}{\partial t} + \frac{R^2}{2} \frac{\partial q}{\partial z} + q \frac{\partial R}{\partial z} = 0 \quad (13)$$

Now, let to introduce the desired variable, the heart chamber volume as:

$$V = \pi R^2 L \quad (14)$$

Where L is the length of each of the heart chambers. And blood flow rate is given as the surface integral of w and $\partial \gamma$ (Labadin and Ahmadi, 2006). Thus:

$$Q = \iint w \partial \gamma = \frac{1}{2} q \pi R^2 \quad (15)$$

From (14) and (15), $\frac{\partial q}{\partial t}, \frac{\partial q}{\partial z}, \frac{\partial R}{\partial t}$ and $\frac{\partial R}{\partial z}$ can be found. After inserting the values of $\frac{\partial q}{\partial t}, \frac{\partial q}{\partial z}, \frac{\partial R}{\partial t}$ and $\frac{\partial R}{\partial z}$ in (12) and (13), another two differential equations are obtained as:

$$\frac{\partial Q}{\partial t} + \frac{3QL}{V} \frac{\partial Q}{\partial z} - \frac{2LQ^2}{V^2} \frac{\partial V}{\partial z} + \frac{4\pi\mu L}{V} Q + \frac{V}{2\rho L} \frac{\partial P}{\partial z} = 0 \quad (16)$$

$$\frac{\partial V}{\partial t} + L \frac{\partial Q}{\partial z} = 0 \quad (17)$$

After combining (16) and (17), a simple differential equation is obtained as follows:

$$\frac{\partial Q}{\partial t} - \frac{3Q}{V} \frac{\partial V}{\partial t} - \frac{2LQ^2}{V^2} \frac{\partial V}{\partial z} + \frac{4\pi\mu L}{V} Q + \frac{V}{2\rho L} \frac{\partial P}{\partial z} = 0 \quad (18)$$

Or, taking $V=SL$, where S is the cross-section area of blood vessels, therefore equation (18) turns to (19):

$$\frac{\partial Q}{\partial t} - \frac{3Q}{S} \frac{\partial S}{\partial t} - \frac{2Q^2}{S^2} \frac{\partial S}{\partial z} + \frac{4\pi\mu}{S} Q + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (19)$$

Equations (18) and (19) have been called as the cardiovascular system equations in terms of volume of heart chamber, and in terms of cross-section area of blood vessels respectively. The model of the heart, blood flow rate, and blood pressure can therefore be derived by applying some logical assumptions issuing by the cardiovascular physics of anatomy and physiology.

The required boundary condition and the values of the other parameters to solve this equation can be obtained from the past works in this field. Such as:

Initial value of $Q=1$ to 5.4 Lmin^{-1} (Liu et al., 1998)

Kinematic viscosity of blood, $\mu=0.035 \text{ cm}^2\text{s}^{-1}$; density of blood, $\rho=1.043$ to 1.057 gcm^{-3} (Hinghofer-Szalkay and Greenleaf, 1987),

$\frac{\partial P}{\partial z}$

pressure gradient, $\partial z = 100$ to 40 mmHg (Labadin and Ahmadi, 2006). Boundary condition for V varies for atria and ventricles, such as: Right ventricle volume = 65 ml and Left ventricle volume = 72 ml (Mansi and Sermesant, 2004), Right atrium volume = 35 ml and Left atrium volume = 40 ml (Barbier, 2009). Heart length = 12 cm (Henry, 1918). If the vertical axis of the heart is considered, then it can be said that each halves contains heart muscle, one atrium and one ventricle. These three portions combine to be 12 cm . If it is assumed that each portion has the equal length, then one third of 12 cm goes for the heart muscle and two third goes for the atrium and ventricle. So, each chamber has a length of 4 cm . Thus, $L=4 \text{ cm}$.

Modeling of the heart

The heart is the center of the cardiovascular system. Remaining within the chest cavity, the heart collects the blood from the different parts of the body, and also pumps the collected blood to the body in a synchronous process provided there is no existence of the heart abnormality. So, the heart can be considered as a point component of the cardiovascular system that is the distance effect on the volume within the heart is vanished. For the same reason, it can be considered that the blood flow rate and the pressure gradient remain constant within the heart's chambers. Applying these assumptions on (18), the cardiovascular system equation reduces to (21), which is the mathematical model of heart:

$$\frac{3Q}{V} \frac{dV}{dt} + \frac{4\pi\mu L}{V} Q + \frac{V}{2\rho L} \frac{\partial P}{\partial z} = 0 \quad (20)$$

$$\Rightarrow \frac{dV}{dt} - \frac{V^2}{6\rho LQ} \frac{\partial P}{\partial z} - \frac{4\pi\mu L}{3} = 0 \quad (21)$$

Modeling of the blood flow rate

Blood flow is the continuous running of blood in the cardiovascular system, and the rate is defined as the amount of blood runs a unit time. To develop the model of blood flow rate, it is assumed that the cross-section area of blood vessel remains unchanged with time, and therefore, the rate of change of cross-section area with respect to distance does not exist. The pressure gradient through the length of blood vessel is also assumed to be constant. Applying these assumptions on (19), the cardiovascular system equation is converted to the model of blood flow rate as depicted in (22):

$$\frac{\partial Q}{\partial t} + \frac{4\pi\mu}{S} Q + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (22)$$

Modeling of the blood pressure

Blood pressure is defined as the pressure experienced upon the walls of blood vessels during the circulation of blood, and this is one of the vital signs of physical fitness. Now, in developing the model of blood pressure, Poisuelli's equation has been used, which determines the relation between blood flow rate and blood pressure, and is given by (23):

$$Q = \frac{\pi R^4}{8L\mu} P \quad (23)$$

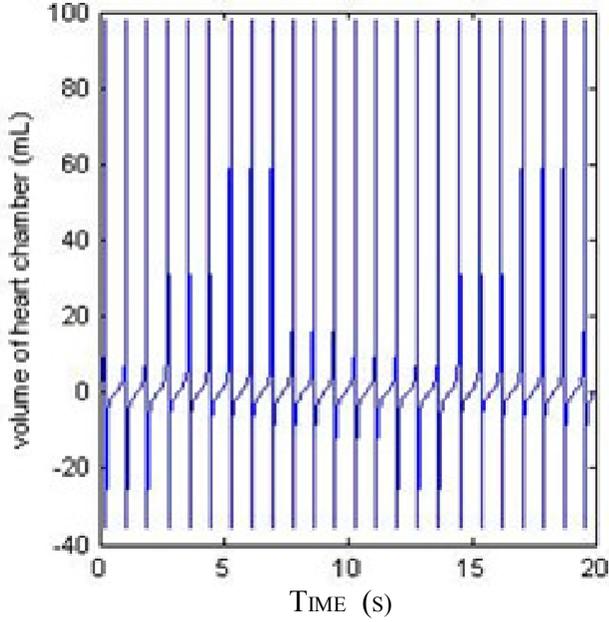


Figure 1. Variation of heart chamber with time.

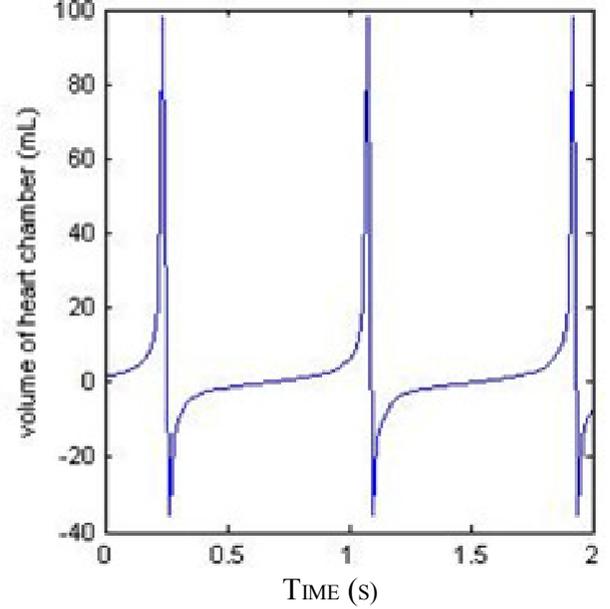


Figure 2. Variation of heart chamber with time.

Where, L is the length and R is the radius of blood vessels. After inserting (23) into (22), the new equations are obtained as follows, and (25) represents the model of blood pressure:

$$\frac{\pi R^4}{8L\mu} \frac{dP}{dt} + \frac{4\pi\mu}{S} \frac{\pi R^4}{8L\mu} P + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (24)$$

$$\Rightarrow \frac{dP}{dt} + \frac{4\mu}{R^2} P + \frac{4L\mu}{\rho R^2} \frac{\partial P}{\partial z} = 0 \quad (25)$$

RESULTS

Analysis of the model of heart

The solution of (21) for the volume of heart chambers provides the characteristics of expansion and contraction of heart. Figure 1 show for longer time while Figure 2, shorter time for large scale view. Both figures show that the volume of the heart chamber increases after some time, which indicates the expansion of the particular heart chamber. After reaching the peak point, it goes to the initial value from the negative direction, which indicates the contraction of the respective heart chamber. Positive direction indicates expansion and consequently, negative direction indicates the contraction of heart chamber. This phenomenon nevertheless is also true in practice and it happens periodically, indicating the heart's rhythm.

Any heart chamber expands and contracts once in a complete heart cycle. So, the time gap between two consecutive peaks of the figures which is 0.84 s, is the

period of heart rhythm. This value is comparable to the actual value (Cromwell et al., 2004).

Response of right ventricle

Equation (21) can be represented as the right ventricle just by applying the initial volume as stated above as the boundary condition. The simulation result is as expected (Figure 3). The numerical values of some parameters obtained are given thus which is comparable as stated in (Mansi and Sermesant, 2004):

End-diastolic volume (EDV) = 114.7939 ml
 End-systolic volume (ESV) = 59.7227 ml
 Stroke volume (SV) = 55.0712 ml
 Ejection fraction (EF) = 47.97%

Response of the right atrium

Equation (21) can be represented as the right atrium just by applying the initial volume as stated above as the boundary condition. The simulation result is as expected (Figure 4). The numerical values of some parameters obtained are given below which is comparable as stated in (Barbier, 2009):

End-diastolic volume (EDV) = 25.3051 ml
 End-systolic volume (ESV) = 56.3052 ml
 Stroke volume (SV) = 31.0001 ml

A trial contraction contributes 31.26% of the total stroke

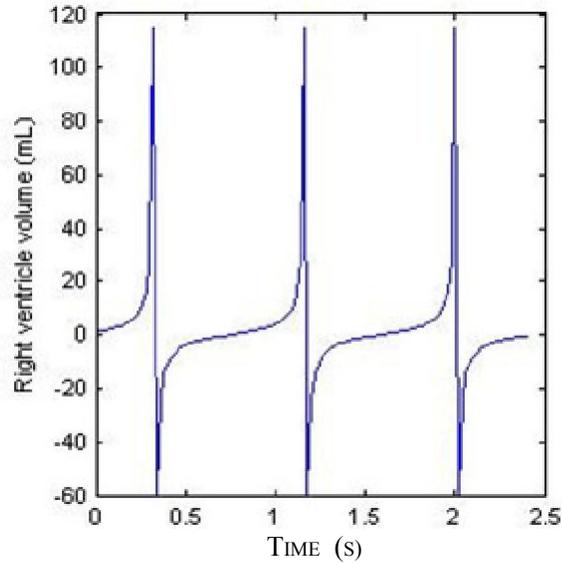


Figure 3. Variation of right ventricle with time.

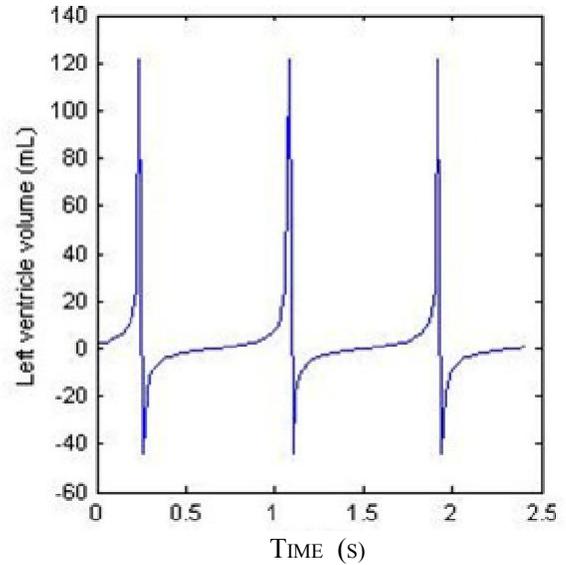


Figure 5. Variation of left ventricle with time.

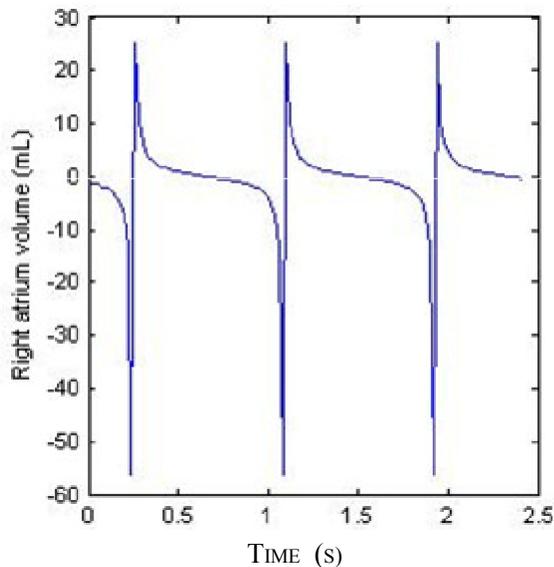


Figure 4. Variation of right atrium with time.

volume which also satisfies as stated in (Karki et al., 2007). As the atrium contracts, the right ventricle expands and vice versa. This opposite relation is shown in Figures 4 and 6 respectively.

Response of left ventricle

Equation (21) can be represented as the left ventricle just by applying the initial volume as stated above as the boundary condition. The simulation was performed which

indicates a rhythmic heart response as expected (Figure 5). The numerical values of some parameters obtained are given below which is comparable as stated in (Mansi and Sermesant, 2004):

End-diastolic volume (EDV) = 121.4316 ml
 End-systolic volume (ESV) = 44.1834 ml
 Stroke volume (SV) = 77.2483 ml
 Ejection fraction (EF) = 63.61%

Response of the left atrium

Equation (21) can be represented as the left atrium just by applying the initial volume as stated above as the boundary condition. The simulation result is as expected (Figure 6). The numerical values of some parameters obtained are given below which is comparable as stated in (Mansi and Sermesant, 2004):

End-diastolic volume (EDV) = 24.2370 ml
 End-systolic volume (ESV) = 62.4113 ml
 Stroke volume (SV) = 38.1744 ml

Atrial contraction contributes 41.29% of the total stroke volume which also satisfies as stated in (Karki et al., 2007).

Heart abnormality analysis

The cardiac abnormality refers to the deviation of cardiac function from its normal behaviors. When the heart faces any abnormalities originated from blockage, leakage etc.

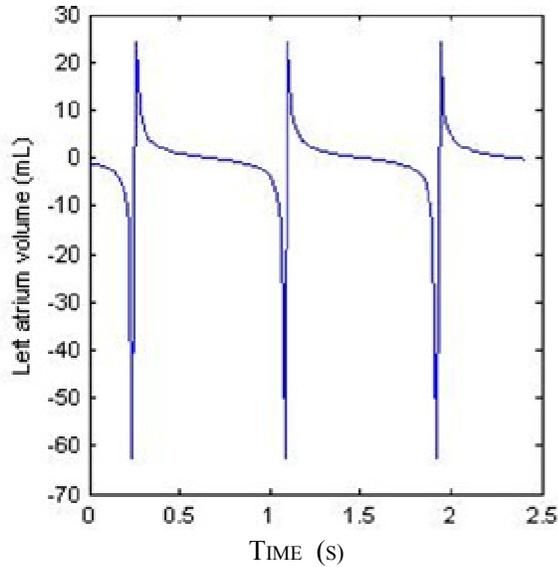


Figure 6. Variation of left atrium with time.

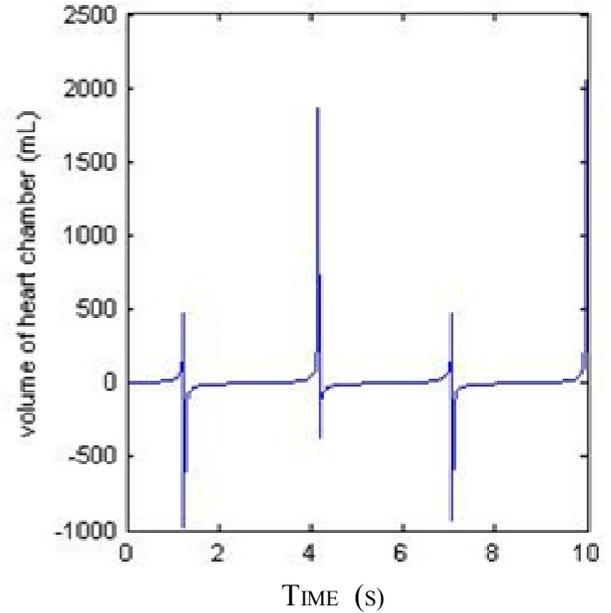


Figure 8. Heart response as blood flow rate decreases extremely.

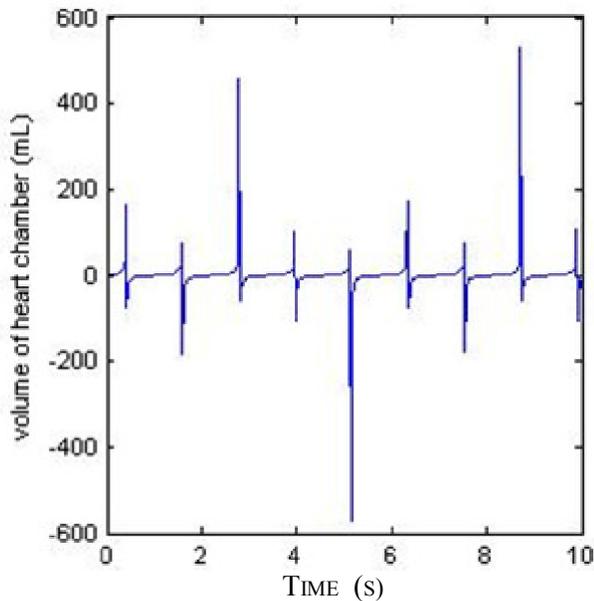


Figure 7. Heart response as blood flow rate decreases a little.

then it is reflected in the blood flow rate and the pressure gradient increment or decrement, which are the two controlling parameters in this model. The amount of increment or decrement depends on the severity of abnormalities.

Decrement in blood flow rate

If the blood flow rate decreases for any abnormality in the

heart, then disorder takes place and heart rate decreases. The simulation result shows that as blood flow rate decreases heart rate also decreases (Figures 7 and 8).

Increment in blood flow rate

If the blood flow rate increases for any abnormality in the heart, disorder takes place and heart rate increases. The simulation result shows this (Figures 9 and 10).

Decrement in pressure gradient

If the pressure gradient decreases for any abnormality in the heart, disorder takes place and heart rate decreases. The simulation result shows that as pressure gradient decreases, heart rate also decreases (Figure 11).

Increment in pressure gradient

If the pressure gradient increases for any abnormality in the heart, disorder takes place and heart rate increases. The simulation result shows that as pressure gradient increases, heart rate also increases (Figure 12).

Heart response with zero flow rate

When blood flow rate is zero, it means that the heart is

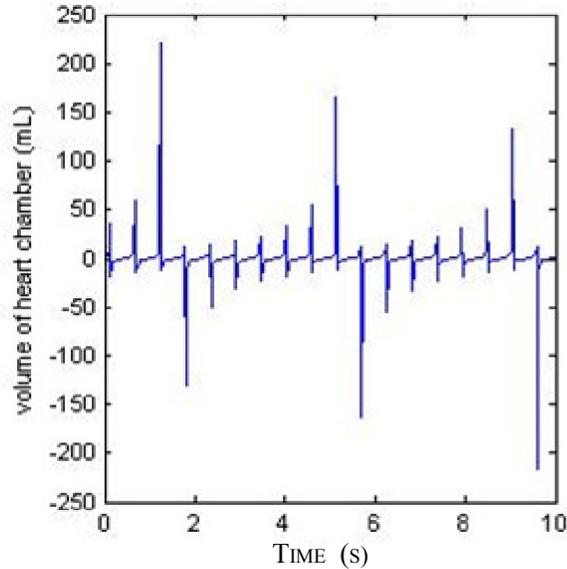


Figure 9. Heart response as blood flow rate increases a little.

totally blocked, and it cannot conduct the blood. Then, heart activity is absent. No blood enters the chamber, so no question to leave the chamber. This implies that the heart neither expands nor contracts. Now, it is time see how this mathematical model handles with the fact that the blood flow rate is equal to zero. The simulation result shows that there is no variation in the volume of heart chamber as the blood flow rate is zero (Figure 13).

Heart response with zero pressure gradient

Zero pressure gradients implies that there is no pressure difference between the two ends of the heart chamber, which is considered as cylindrical shaped in this study. If there is no pressure difference, then the blood does not pass through the heart chamber. So, somehow, the blood entering into the chamber does not leave it. Thus, the volume will increase proportionally. Now, it is to be seen what this model says. The simulation result of this model is plotted. Figure 14 illustrates that the volume of the heart chamber increases with time as the pressure gradient is kept at zero.

Analysis of the model of blood flow rate

According to the Poisuelli's equation, the flow rate increases with the cross-section area. The model of blood flow rate presented here also shows that the blood flow rate inside the body varies accordingly with the cross-section area of the blood vessels (Figure 15).

Pressure gradient is also directly related to blood flow rate, and accordingly, pressure is higher at the beginning

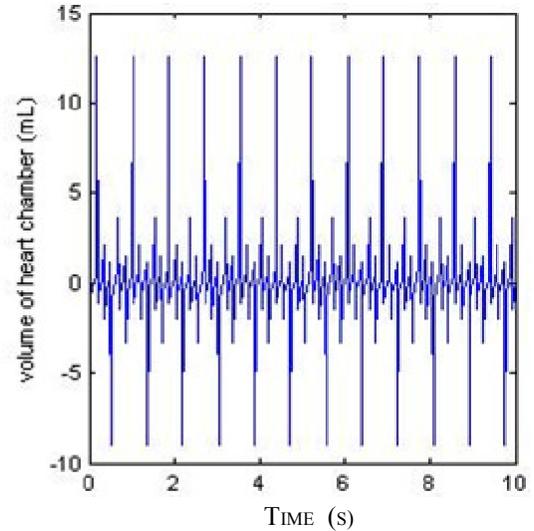


Figure 10. Heart response as blood flow rate increases extremely.

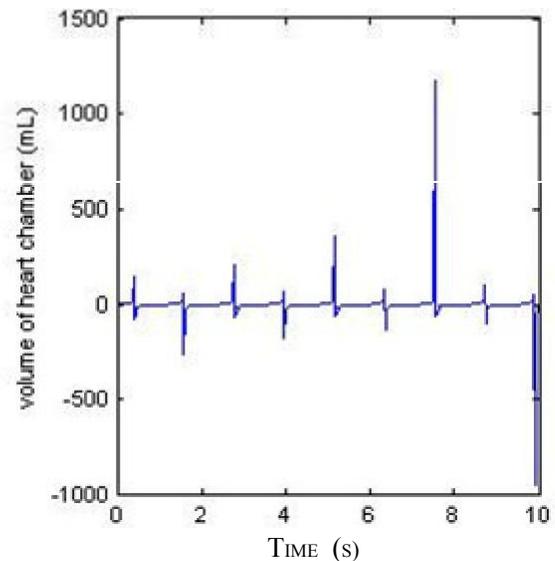


Figure 11. Heart response as pressure gradient decreases extremely.

than at the end of vessel, establishing a pressure gradient. The greater the pressure gradient forcing blood through a vessel, the greater the rate of flow through the vessel (Sherwood, 2005). The proposed model also shows that for a given pressure gradient, the blood flow rate decreases with time, and vice versa (Figure 16).

Analysis of the model of blood pressure

The solution for blood pressure in the corresponding

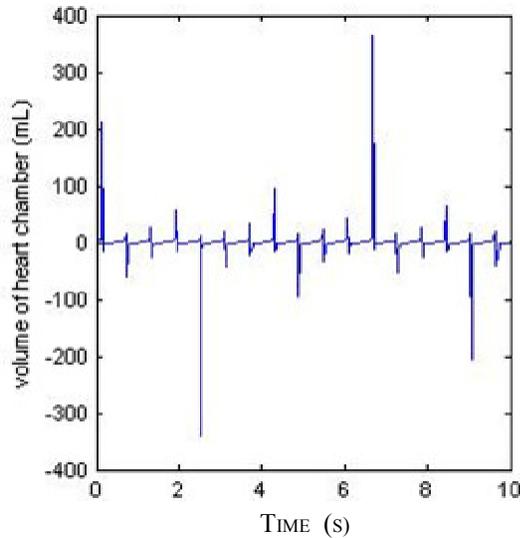


Figure 12. Heart response as blood flow rate increases extremely.

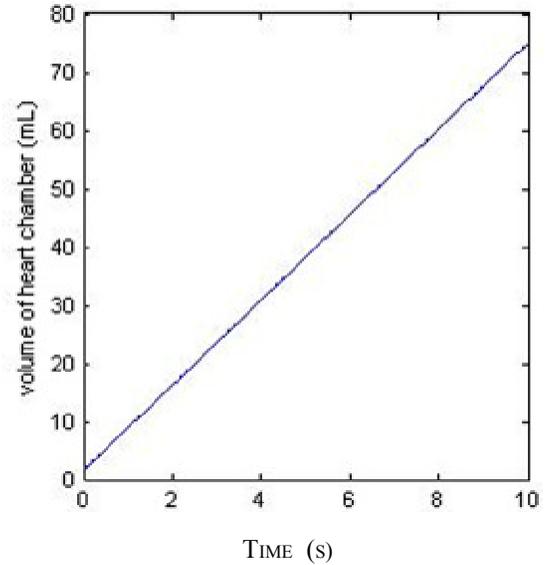


Figure 14. Heart chamber volume at zero pressure gradients.

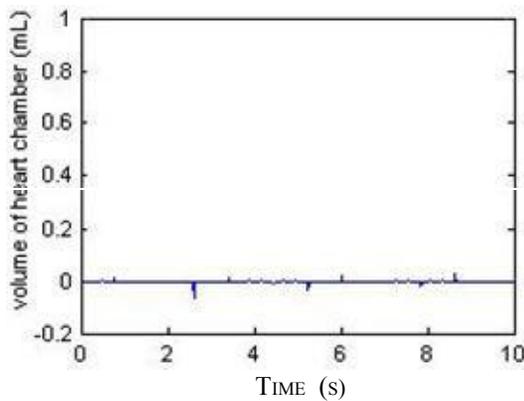


Figure 13. Heart chamber volume at zero blood flow rate.

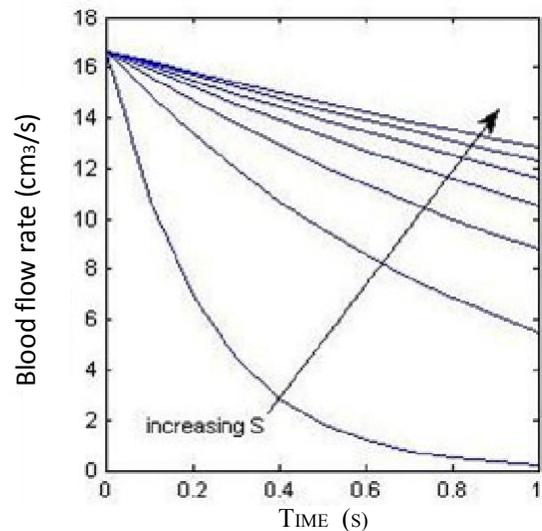


Figure 15. Blood flow rate for different cross-sectional area (from 0.1 to 2 cm²).

model represents blood pressure decreases with the cross-sectional area (Figure 17). This result is supported by the Poiseuille's equation, the fundamental equation of fluid dynamics, which states that the pressure of fluid flow is inversely proportional to the cross-section area of the fluid conveyance provided that others related parameters remain constant.

Again, from the Poiseuille's equation, it can be concluded that the pressure on the wall of fluid conveyance is directly proportional to its length. This statement has also been derived from the model of blood pressure presented in this research, which indicates the increment of blood pressure with the increment of length of the blood vessels (Figure 18). Higher pressure at the beginning and lower pressure at the end and the difference between this two, highly varies with length of

the blood vessels.

Systolic (maximum) blood pressure in the normal adult is in the range of 95 to 140 mmHg, with 120 mmHg being average. These figures are subject to much variation with age, climate, eating habits, and other factors. Normal diastolic blood pressure (lowest pressure between beats) ranges from 60 to 90 mmHg; 80 mmHg being about average. This pressure is usually measured in the brachial artery in the arm (Cromwell et al., 2004). These models support the normal ranges (Figures 17 and 18).

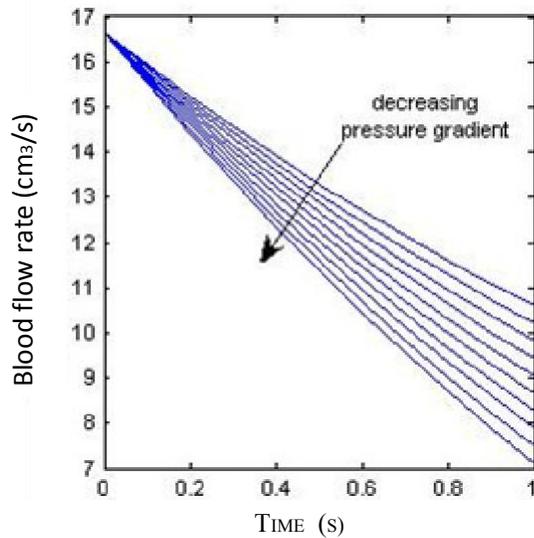


Figure 16. Blood flow rate for different pressure gradients (from 40 to 100 mmHg).

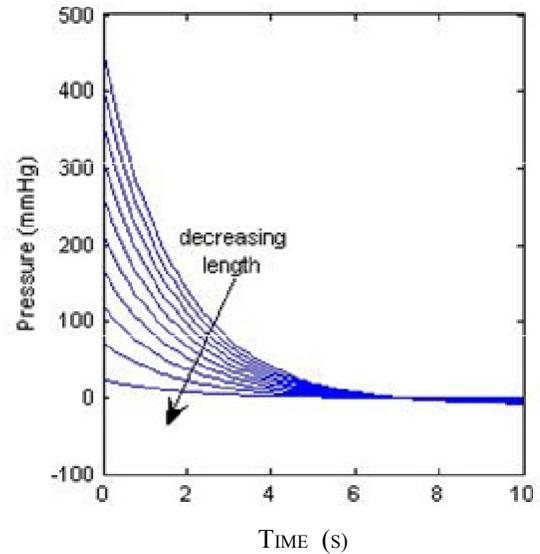


Figure 18. Blood pressure for different length of vessels.

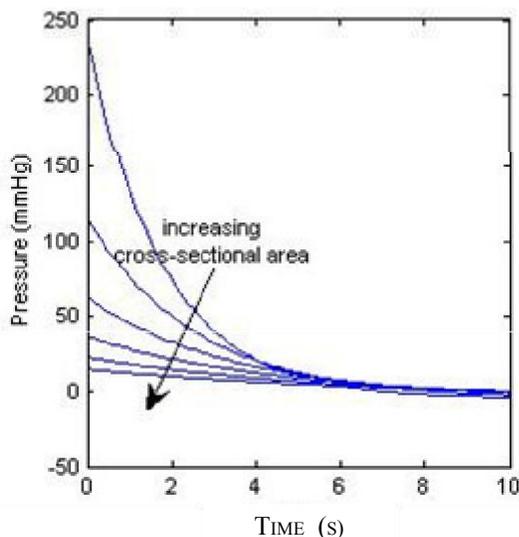


Figure 17. Blood pressure for different cross-sectional area of vessels.

Conclusion

The design of computational models of human organs is a new research field, which opens new possibilities for medical image analysis and therapy simulation (Sermesant et al., 2006). Therefore, a little effort was given in the purpose of developing a mathematical model of cardiovascular system that can describe the entire cardiac activities.

A limited number of internal parameters were

considered in developing the model. So, possible improvements of the study would include the integration of more anatomical structure (valve, exact size of the heart chambers), more realistic model and a more complex constitutive law. However, the objective of this research was not to build the more complex and faithful model ever.

Although a large number of assumptions have been considered, the model can still be treated as valid, because this model is able to show the periodicity in the expansion and contraction of the heart chambers, can represent each of the heart chambers only by changing the initial conditions, and can also represent the different types of heart abnormalities such as blockage, leakage etc. for which the blood flow rate and the pressure gradient changes. It is shown that in case of any heart abnormalities, the normal heart rhythm is lost, and a disorder takes place. The models of blood flow rate and blood pressure are capable to satisfy the fundamental principles of fluid dynamics. All of these analyses prove nothing but the validity of the model.

Furthermore, the mathematical model of heart can be used to determine the heart rate from the periodicity property (Figure 1). The ability to represent the heart abnormality of this model helps to determine the type of heart diseases. Moreover, the blood flow rate and the blood pressure of any particular person can be measured using the respective models.

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