

*Full Length Research Paper*

# Another look at Kostiakov, modified Kostiakov and revised modified Kostiakov infiltration models in water resources applications

Prabeer Kumar Parhi

Center for Water Engineering and Management, Central University of Jharkhand, Ranchi-835205, India.  
E-mail: [prabeer11@yahoo.co.in](mailto:prabeer11@yahoo.co.in)

Accepted 17 January, 2014

Owing to their simplicity and yielding reasonably satisfactory results in most applications, Kostiakov (KT) and modified Kostiakov (MKT) empirical infiltration models have been quite popular and frequently used in various water resource applications world over. With an improvement to the modified Kostiakov model known as revised modified Kostiakov (RMKT) model, there has been a need to critically analyze and investigate all the three popular empirical models to arrive at some conclusions regarding their suitability of application. The present study allows to conclude that except for Yolo light clay, where the performance was similar to the one of MKT, RMKT model performs better than MKT and KT models for the Plain field sand, Columbia sandy loam and Narsingpur Clay, with a very significant improvement in performance for Columbia sandy loam. Hence the use of RMKT model is strongly recommended for Columbia sandy loam type of soils and soils which are not fully saturated whilst MKT model can be recommended for the soils which are completely saturated with water.

**Key words:** Empirical infiltration models, Kostiakov model, modified Kostiakov model, revised modified Kostiakov model, least square approach, Nash-Sutcliffe efficiency.

## INTRODUCTION

In the irrigation and drainage engineering as well as in surface and sub-surface hydrology, the role of infiltration is very important. The rate of infiltration at a time describes the capacity of a soil to absorb water, which enables the determination of soil moisture status. After accounting for the losses, in which infiltration plays a very critical role, the remaining rainfall-excess leads to a runoff generation. It is due to these reasons that the infiltration has received a great deal of attention from soil and water scientists and large numbers of models for computation of infiltration have been developed. A robust infiltration model, that can correctly predict the actual infiltration, can be quite effective in planning and designing of water resources systems.

Infiltration models can generally be classified into three groups (Mishra and Singh, 1999) (i) physically based models, (ii) semi-empirical and (iii) empirical. Physically based models rely on the law of mass conservation and the Darcy law. Depending on the considerations of flow dynamics, hydraulic conductivity, moisture content, initial and boundary conditions, physically based models of

varying complexity have been derived. Among the physically based models, the models of (Green and Ampt, 1911; Phillip, 1957, 1969; Smith, 1972), are worth citing. Semi-empirical models employ simple forms of continuity equation and simple hypothesis on the infiltration rate-cumulative infiltration. These models are based on the systems approach, popularly employed in surface water hydrology. Examples of semi-empirical models are the models of (Horton, 1938; Overton, 1964; Singh and Yu, 1990). Empirical models are derived from data observed either in field or in laboratory. The models of Kostiakov (1932); Huggins and Monke (1966) modified Kostiakov, (Smith, 1972) and revised modified Kostiakov (Parhi et al., 2007) fall in the category of empirical models.

Despite the availability of a large number of infiltration models, these are still being either developed or modified. The recent version of Soil Conservation Service Curve Number (SCS-CN)-based infiltration model is only an example (Mishra and Singh, 2003). Owing to their simplicity and yielding reasonably satisfactory results in

most applications, some of the available empirical models have been quite popular and frequently used in various water resource applications world over. The wide-spread application of the Kostiakov (KT) and the modified Kostiakov (MKT) models in irrigation engineering are examples worth citing (Michael, 1982). Recently Parhi et al. (2007), revised the Kostiakov (KT) and the modified Kostiakov (MKT) infiltration model and developed a four parameter infiltration model known as revised modified Kostiakov (RMKT) model which shows significant improvement over the former models on silt, clay loam and silt clay loam soils. Very recently, Zolfaghari et al. (2012) compared various models for estimating cumulative infiltration and observed that the RMKT model performs best on silt, clay loam and silt clay loam soils.

In the above context, there exists a lot of scope to further analyze and investigate the KT, MKT and RMKT infiltration models, as these models are very popular and widely used for computation of infiltration. Thus, the objective of the present study was to critically analyze above empirical infiltration models (KT, MKT and RMKT) and to verify their performance on a large set of infiltration data derived from different soils and to suggest their suitability.

## Theory of Infiltration

When water is added to a dry soil either by rain or irrigation, it is distributed around the soil particles where it is held by adhesive or cohesive forces. It gradually displaces air in the pore spaces and eventually fills the pores. When all the pores, large and small, are filled, the soil is said to be saturated and it is at its maximum retentive capacity. The time at which the capillary potential ( $\psi$ ) approaches zero, the beginning of runoff and decay of infiltration rate begins. This time to ponding ( $t_p$ ) (Smith, 1972; Mishra and Singh, 2003) is a function of the initial moisture content of the soil. The more the soil moisture content the less is the time needed to ponding. In addition, ' $t_p$ ' shows a strong dependence on the rainfall

rate (R) as  $t_p = a_1 R^{a_2}$  (Smith (1972), where ' $a_1$ ' and ' $a_2$ ' are constants, which depend on the type of soil and initial moisture content. Under a given antecedent moisture condition, a soil exhibits the maximum rate of infiltration at time = 0 (initial), which decreases as more water infiltrates into the soil with increasing time and finally achieves almost a constant rate known as ultimate infiltration capacity ( $f_c$ ). This theory influences and describes the general behavior of infiltration model under a uniform rainfall or the irrigation rate for all types of soils.

## Kostiakov Infiltration Model

The popular Kostiakov (KT) and modified Kostiakov (MKT)

infiltration models frequently used in irrigation engineering are derived using the data observed in either the field or the laboratory. According to Kostiakov 1932, the best-fit curve of infiltration appears to be

$$f = \alpha t^\beta \dots\dots\dots (1)$$

where  $\alpha$  is the coefficient and  $\beta$  is the exponent and  $t$  is the time period over which infiltration process continues. Both the parameters rely on soil type, initial moisture content, rainfall rate, vegetative cover. The values of parameters are determined experimentally.

## Modified Kostiakov (MKT) Infiltration Model

Smith (1972) modified the KT equation by including the term ' $f_c$ '. The logic for the inclusion of ' $f_c$ ' is that the infiltration rate decreases as more water infiltrates into soil until a constant rate known as ultimate infiltration capacity is achieved. According to Smith (1972), the best-fit curve of infiltration is given as:

$$f = f_c + \alpha_1 t^{-\beta_1} \dots\dots\dots (2)$$

Where  $\alpha_1$  is the coefficient and  $\beta_1$  is the exponent. Both the parameters rely on soil type, initial moisture content, rainfall rate, vegetative cover and their values are determined experimentally.

## Revised Modified Kostiakov (RMKT) Model

Parhi et al. (2007) suggested that at the beginning, the infiltration rate is maximum, known as initial infiltration rate ( $f_0$ ). As more water is added, the soil moisture potential decreases and infiltration rate starts decaying until the soil moisture potential or capillary potential reaches a value of one-third atmospheres. At this point, all the macro pores of soil are filled with water and soil is said to be at field capacity. When soil moisture potential falls below one-third atmospheres and approaches zero, all the pores of soil get filled with water and infiltration rate reaches a constant value, known as ultimate infiltration capacity ( $f_c$ ). Thus ' $f_c$ ' increases from zero to a maximum value, beyond which it remains constant. From the above rationale, Parhi et al. (2007) suggested that the term ' $f_c$ ' can be best expressed as a time varying function, in a rising power form expressed as:

$$f_c = \alpha_2 t^{-\beta_2} \dots\dots\dots (3)$$

Hence they modified the MKT equation in the revised form as:

$$f = \alpha_2 t^{\beta_2} + \alpha_3 t^{-\beta_3} \dots\dots\dots (4)$$

where ' $\alpha_2 t^{\beta_2}$ ' represents that time varying infiltration component which after complete saturation of the soil represents the ultimate infiltration capacity of the soil and the term ' $\alpha_3 t^{-\beta_3}$ ' represents the continuously decaying dynamic infiltration component. When added, the resulting infiltration rate corresponds to the infiltration capacity curve. The coefficients  $\alpha_2$ ,  $\beta_2$  and exponents  $\alpha_3$ ,  $\beta_3$  are can be determined from field data using an appro-

**Table 1.** Infiltration Data

SI. No.	Soil.	Region	Country	Datasets used for simulation	Datasets used for model calibration	Dataset s used for model testing	$K_s$ (cm s <sup>-1</sup> )	Porosity	Reference
1	PFS (disturbed sample)	Minnesota	USA	7	7	5	$3.44 \times 10^{-3}$	0.477	Black et al. 1969
2	CSL (disturbed sample)	Minnesota	USA	9	9	4	$1.39 \times 10^{-3}$	0.518	Laliberte et al. 1966
3	YLC (disturbed sample)	Minnesota	USA	4	4	2	$1.23 \times 10^{-5}$	0.499	Moore (1939)
4	Narsinghpur Clay (NC)	Madhya Pradesh	India	6	6	2			Roy and Singh, 1995
5	Cultivated field (Crop height 65 cm)		India	1					Michael 1982

appropriate optimization technique, such as the least squares approach.

## MATERIALS AND METHODS

### Infiltration Data

The data employed in this study were obtained from infiltration tests carried out in the laboratory and field in USA and India. The infiltration data for Plainfield Sand (PFS), Columbia Sandy Loam (CSL) and Yolo Light Clay (YLC) were obtained from several laboratory tests reported by (Mein and Larson, 1971). The infiltration tests carried out in India were from Sher basin falling in the Narisinghpur district of Madhya Pradesh (Roy and Singh, 1995) represents Narisinghpur Clay (NC) and the cylinder infiltrometer test data were reported by (Michael 1982). The infiltration data of Michael were from cultivated fields with a crop height of 65 cm and soil moisture content of 4.87 %. The details of data used in this study are shown in Table 1.

### Parameter Estimation

Optimal values of parameters of the above mentioned models were estimated using the method of least squares, a device for finding the equation of a specified type of curves, which best fits a set of observations. The

method suggests that, for the best fits, the sum of the squares of differences between the observed and the corresponding estimated values is minimum (Parhi et al., 2007). In this study, the software package Language for Interactive General Optimizer (LINGO) was used to minimize the errors as:

$$\text{Min } Z = \sum_{i=1}^N \{f_{obs}(i) - f_{comp}(i)\}^2 \quad \dots (5)$$

where, Z = error; N = the number of observations or times;  $f_{obs}(i)$  = observed infiltration rate at  $i^{\text{th}}$  time;  $f_{com}(i)$  = computed infiltration rate at  $i^{\text{th}}$  time. Table 2 shows the computed minimum, maximum and average values of model parameters resulting from various model applications to different infiltration data sets.

### Performance Evaluation

Among the several statistical measures available for evaluating the performance of a model, such as correlation coefficient, relative error, standard error, the (Nash and Sutcliffe, 1970) efficiency is most frequently used (Mishra and Singh, 2003) and it has been employed in this study. It is expressed in percent form as:

$$\text{Efficiency} = (1 - D_1/D_0) * 100 \dots \dots \dots (6)$$

where,  $D_1$  is the sum of square of deviations/differences between computed and observed data, expressed as:

$$D_1 = \sum (Y_0 - Y_1)^2 \dots \dots \dots (7)$$

**Table 2.** Statistics of optimized parameters on various soil types

Soil	Statistics	KT		MKT			RMKT			
		A	$\beta$	fc	$\alpha_1$	$\beta_1$	$\alpha_2$	$\beta_2$	$\alpha_3$	$\beta_3$
PFS	Minimum	50.49	0.44	0	29.13	0.51	0	0	29.13	0.51
	Maximum	59.09	0.55	17.17	54.49	1.02	17.17	0.11	54.49	1.02
	Average	53.59	0.50	15.37	37.34	0.87	14.43	0.02	38.30	0.86
CSL	Minimum	44.14	0.49	5.74	51.78	0.81	0.51	0	63.55	0.65
	Maximum	70.19	0.54	8.44	175.16	1.16	8.18	0.54	168.23	1.13
	Average	54.19	0.50	6.93	89.51	0.93	5.85	0.09	88.74	0.89
YLC	Minimum	5.39	0.63	0.04	8.86	0.77	0.04	0	8.76	0.77
	Maximum	9.78	0.69	0.07	55.37	1.21	0.07	0	55.37	1.06
	Average	7.05	0.65	0.06	30.60	0.976	0.063	0	30.607	0.976
NC	Minimum	3.015	0.323	0	3.015	0.439	0	0	3.015	0.439
	Maximum	69.02	0.909	0.376	69.020	1.03	0.376	0	69.02	1.03
	Average	25.24	0.6708	0.1076	27.446	0.715	0.1076	0	27.446	0.715

4

**Table 3.** Simulation of infiltration data on various types of soil

Soil Type	Sl. No	KT	MKT	RMKT
PFS	1	98.348	98.923	99.923
	2	97.263	99.787	99.787
	3	96.781	98.569	98.569
	4	98.290	99.229	99.230
	5	98.380	99.801	99.805
	6	97.061	99.757	99.759
	7	96.530	99.695	99.696
	AVG	97.522	99.537	99.539
CSL	1	97.200	97.742	98.534
	2	98.432	99.897	99.897
	3	99.366	99.844	99.844
	4	99.441	99.958	99.958
	5	99.390	99.563	99.584
	6	98.113	99.706	99.706
	7	99.424	99.982	99.986
	8	99.030	99.777	99.779
	9	97.869	99.589	99.589
AVG	98.696	99.562	99.653	
YLC	1	98.27	98.555	98.555
	2	99.791	99.907	99.907
	3	98.460	99.275	99.275
	4	92.455	97.448	97.448
	AVG	97.244	98.796	98.796
NC	1	95.907	95.907	95.907
	2	97.296	97.996	97.996
	3	92.695	92.695	92.695
	4	93.000	93.000	93.000
	5	86.745	87.120	87.120
	6	98.326	98.326	98.326
	AVG	93.995	94.168	94.168

**Table 4.** Models performance (validated) on various soils

Soil Type	Efficiency Statistics	KT	MKT	RMKT
PFS	Minimum	8.05	19.81	20.20
	Maximum	92.24	98.99	99.04
	Average	59.532	63.302	63.470
CSL	Minimum	28.15	41.39	85.54
	Maximum	72.22	83.47	96.54
	Average	50.515	62.595	91.900
YLC	Minimum	32.98	60.3	60.3
	Maximum	41.14	67.26	67.26
	Average	37.56	63.78	63.78
NC	Minimum	53.70	54.752	54.752
	Maximum	81.935	82.018	82.018
	Average	67.8175	68.385	68.385

and  $D_0$  is the initial variance, which is the sum of the square deviations between the observed data about the observed mean, expressed as:

$$D_0 = \frac{1}{n} \sum (Y_0 - Y_m)^2 \dots \dots \dots (8)$$

Where,  $Y_0$  = observed data;  $Y_1$  = computed data;  $Y_m$  = mean of observed data.

## RESULTS AND DISCUSSION

### Calibration of Model Parameters and Simulation using Datasets Used for Calibration

To calibrate the model parameters 27 infiltration datasets of five different soils (Table 1) were considered. The values of the infiltration parameters (minimum, maximum and average) were computed using the method of the least squares as described above.

Using the average values of the computed parameters (Table 2) for the different types of soils, the ability of above infiltration models was evaluated in terms of Nash and Sutcliffe efficiency and the performance results are shown in Table 3. It can be observed that, in the case of PFS, both RMKT and MKT showed an enhanced performance over KT on all datasets and RMKT over MKT on 4 datasets.

Similarly, for all the 9 datasets of CSL, the enhanced performance was visible for both RMKT and MKT on all data sets and RMKT over MKT on 4 datasets. However, the enhanced performance of RMKT over MKT was very significant.

In the cases of YLC and NC, 4 and 6 samples were respectively considered.

For YLC, the performance of RMKT and MKT models was similar, but both showed enhanced performance over KT in all datasets.

On 2 datasets of NC, RMKT performed same as MKT. On all other datasets, all the models performed equally well.

### Simulation of Infiltration Data on Various Soils for Validation

For the model validation, 13 datasets of different soils which were not used for calibration were considered (column 6 of Table 1). After employing the average parameter values (Table 2), the ability of the above infiltration models to simulate infiltration process was validated and their maximum, minimum and average values were computed (Table 4). The model performance was compared using the average values of these efficiencies on different data sets. On PFS, CSL and NC data, KT performed the poorest, MKT better than KT and RMKT performed best. However, on YLC data, MKT performed better than KT and both RMKT and MKT performed almost equally.

## CONCLUSIONS

The following conclusions can be derived from the present study:

1. On PFC, CSL, and NC, RMKT model performs better than MKT and KT model.
2. RMKT model shows significant improvement in performance over both MKT and KT on Columbia Sandy Loam (CSL). Hence use of RMKT model is strongly recommended for CSL type of soils.
3. Irrespective of the type of soil both RMKT and MKT models perform better than KT model. However, on YLC RMKT and MKT model performs equally.
4. As a general practice, use of MKT and RMKT models can be recommended respectively on completely saturated and unsaturated soils.

## REFERENCES

Black TA, Gardner WR, Thurtell GW (1969). The prediction

- of evaporation, drainage and soil water storage for a bare soil. *Soil Sci. Soc. Am.* 33: 655-660.
- Green WH, Ampt GA (1911). Studies on soil physics I. Flow of air and water through soils. *J. Agric. Sci.* 4: 1-24.
- Horton RL (1938). The interpretation and application of runoff plot experiments with reference to soil erosion problems. *Soil Sci. Soc. Am.* 3: 340-349.
- Huggins LF, Monke EI (1966). The mathematical simulation of the hydrology of small watersheds." Technical Report No. 1 Purdue Water Resources Research Centre, Lafayette.
- Kostiakov AN (1932). On the dynamics of the coefficient of water-percolation in soils and on the necessity of studying it from a dynamic point of view for purposes of amelioration. *Proceedings of the 6th Congress of International Soil Science Society Transactions, Moscow, (ISSST'32), Part A: 17-21.*
- Laliberte GE, Corey AT, Brooks RH (1966). Properties of unsaturated porous media. *Hydrology Paper No. 17, Colorado State University, Fort Collins.*
- Mein RG, Larson CL (1971). Modeling the Infiltration Component of the Rainfall-Runoff Process. *WRRRC Bull. No. 43, Water Resources Research Center, University of Minnesota, Minneapolis.*
- Mishra SK, Singh VP (1999). Another look at the SCS-CN method. *J. Hydrol. Eng.* 4: 257-264.
- Michael AM (1982). *Irrigation Theory and Practices.* Orient Long Men Co., New Delhi, p. 469.
- Mishra SK, Singh VP (2003). *Soil Conservation Service Curve Number (SCS-CN) Methodology.* 1st Edn. Kluwer Academic Publishers, Dordrecht, The Netherlands, ISBN-13: 978-1402011320.
- Moore RE (1939). Water conduction from shallow water tables. *Hilgardia*, 12: 383-426.
- Nash JE, Sutcliffe JV (1970). River flow forecasting through conceptual models, Part I-A discussion of principles. *J. Hydrol.* 10: 282-290.
- Overton DE (1964). Mathematical refinement of an infiltration equation for watershed engineering. *ARS 41-49, US Department of Agricultural Services, Washington, DC., USA.*
- Parhi PK, Mishra SK, Singh R (2007). A Modification to Kostiakov and Modified Kostiakov Infiltration Model, *Water Resources Management, Kluwer Academic Publishers* 21: 1973-1989.
- Phillip JR (1957). Theory of infiltration. *Soil Science*, 83 (5): 345-357.
- Phillip JR (1969). *Theory of Infiltration: Advances in Hydroscience, Chow V.T. (Ed.). Academic Press, New York, USA., 215-296.*
- Roy BP, Singh H (1995). Infiltration study of a sub basin. Case Study CS (AR)-170, National Institute of Hydrology, Roorkee, India.
- Smith RE (1972). The infiltration envelope: results from a theoretical infiltrometer. *J. Hydrol.* 17: 1-21.
- Singh VP, Yu FX (1990). Derivation of infiltration equation using systems approach. *J. Irrigation Drainage Eng.* 116: 837-857.
- Zolfaghari AA, Mirzaee SM, Gorzi M (2012). Comparison of different models for estimating cumulative infiltration. *Int. J. Soil Sci.* 7: 108-115.