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Review

A note on hierarchy evolutionary game

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Stochastic evolutionary game dynamics in hierarchy are very popular in economics and in management sciences. This study extends the model of stochastic evolutionary game dynamics. This paper highlights hierarchy selection model and we show that the hierarchy selection games are more stable without structures in this paper. Furthermore, an example in industrial organization is given to rationally explain the theory in this work.

Key words: Fitness, fixation probability, invasion, game theory, economics, hierarchy, evolutionary game.

INTRODUCTION

Evolutionary dynamics, which have been traditionally studied in biological field and are recently focused on, are considered with finite populations (Liebermat et al., 2005; Taylor et al., 2004; He and Cui, 2007). On the long run, all social and natural phenomena are actually dynamic, and evolutionary dynamic is consequently an exceedingly powerful tool to analyze them. In Liebermat et al. (2005), Taylor et al. (2004) and Ma (2004) and the references mentioned therein, selections in finite populations are modeled as evolutionary dynamic games. Game theory, as an extremely important branch in applied mathematics (Fudenberg and Tirole, 2003; Nie, 2005, 2009, 2010) is pervaded almost all fields, and is strongly extended to evolutionary dynamic situations. Selection on multiple levels are recently studied with evolutionary dynamics idea (Traulsen et al., 2005). In Nie (2007), evolutionary dynamic games are extended to economic fields and some economic phenomena are rationally explained. In Nie (2007), the difficulties for a firm to enter an industry, or the fitness of the selection games, are analyzed. When a firm enters an industry, this firm may escape this field because of low fixation. The fixation probability is accordingly important and is considered in this paper and some economic phenomena are rationally explained. Many economic and social organizations are hierarchy structures. In an organization, for example, there are operatives and managers (including first-line managers, middle managers and top managers in hierarchy) (Robbins and Decenzo, 2002), which consists in a hierarchy structure. Moreover, bureaucratic control is also a hierarchy structure (Robbins and Decenzo, 2002; Whitman, 2005). More recently, work on both human and non-human primates has suggested that social groups are often hierarchically structured and the social group sizes are considered (Zhang and Nie, 2010; Zhou et al., 2005).

For political institution, in China there exist national government, provinces, counties and so on in hierarchy and the higher rank government (mainly) determines the lower rank governments. It is therefore a multi-level problem. In European Union, there also exists a multi-level political institution (Bache and Flinder, 2004). In summary, it is crucial to consider hierarchy structures. This paper is organized as follows: a type of selection games in Taylor et al. (2004), which is extended to hierarchy structure, is analyzed and extended to economic field. The fixation probability of selection games in hierarchy structure is considered. Some remarks are given in the finally.

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The fixation probability of selection games

We here introduce the model of selection games in economics as follows: which are also introduced in Nie (2007). When a firm hopes to enter some industry, in general this firm has to fully consider the possibility. This firm should accordingly consider the benefit and the cost before he hopes to invade the corresponding industry. On the other hand, the other firms which have been in this industry, may accept or reject this firm to enter the

Table 1. The corresponding payoff matrix for game.

	A	B
A	a	b
B	c	d

industry according to their benefits. It is consequently a game between this firm and the other firms. We now consider these firms. We assume the firms in this industry are identical to simplify the problem. We consider the evolutionary dynamics of a game with two strategies *A* and *B*, meaning to accept and to reject, respectively. The model is the same as that in the paper (2007). The corresponding payoff matrix for this game is given by Table 1. Strategy *A* player receives payoff *a* when playing against another strategy *A* player, and payoff *c* when playing against a strategy *B* player. *A* strategy *B* player would receive payoff *b* and *d* when playing against *A* and *B* players, respectively. Similar to that in the paper of Taylor et al. (2004), we also denote p_A and p_B the frequency of individuals employing strategy *A* and *B*, respectively, and $p_A + p_B = 1$, where $p_A \geq 0$ and $p_B \geq 0$. We also define the fitness similar to that in Liebermat et al. (2005), Taylor et al. (2004) and Traulsen et al. (2005), and assume the fitnesses of *A* and *B* are given by:

$$f_A = ap_A + bp_B$$
$$f_B = cp_A + dp_B . \quad (1)$$

The model is the following replicator equations as follows:

$$\dot{p}_A = p_A(f_A - \phi),$$
$$\dot{p}_B = p_B(f_B - \phi). \quad (2)$$

Where ϕ is the average fitness given by:

$$\phi = f_A p_A + f_B p_B . \quad (3)$$

The fixation probability $\rho_A(\omega, N)$ is the probability that a single individual A will invade and take over a population of *N* B players. About the fixation probability, we have the following results with the similar technique in Nowak et al. (2004), which are also given in Taylor et al. (2004).

Corollary 1

If i is the number of individuals employing strategy A and $N - i$ is the number of individuals employing strategy B, ω denotes the fitness in this game with the strategy A, then:

$$f_i = 1 - \omega + \omega[a(i-1) + b(N-i)]/(N-1),$$
$$g_i = 1 - \omega + \omega[c(i-1) + d(N-i)]/(N-1). \quad (4)$$

Furthermore,

$$\rho_A(\omega, N) = \frac{1}{1 + \sum_{k=1}^{N-1} \prod_{i=1}^{k} \frac{g_i}{f_i}} . \quad (5)$$

Note

When $\rho_A(\omega, N) > \frac{1}{N}$, selection favors *A* replacing *B*. There exist various instances in economics in practice to support the aforementioned theory.

We now consider the stability of the evolutionary dynamic games on two levels. Assume that there is a game between two groups, one with *n* individuals and the other with *m* individuals. Furthermore, these two groups play in the different position or in hierarchies. One group lies in the upper level and the other group lies in the lower level. Denote the corresponding fixation probability to be $\phi(\omega, m, n)$ for the hierarchy selection with *m* individuals in the upper level and *n* individuals in the lower level. We then have the following result:

Theorem 1

For the evolutionary dynamic games between two groups on two levels, one with n individuals and the other with m individuals, the fixation probability is:

$$\phi(\omega, m, n) = \rho_A(\omega, m)\rho_A(\omega, n) \quad (6)$$

Proof: Consider the fixation probability of a single mutant in the group with *m* individuals. This mutant firstly has to reach fixation in its group, which induces $\rho_A(\omega, m)$. This group then has to overwhelm other group and $\rho_A(\omega, n)$ is obtained. The fixation probability is therefore $\phi(\omega, m, n) = \rho_A(\omega, m)\rho_A(\omega, n)$. The result is accordingly obtained immediately and the proof is complete. We now compare the aforementioned two-level game with the game of *m+n* individuals and the following result is obtained.

Theorem 2

For the evolutionary dynamic games between two groups on two levels, one with n individuals and the other with m individuals, if a > c and b > d, then, the fixation probability satisfies:

$$\phi(\omega, m, n) \le \rho_A(\omega, m+n) \tag{7}$$

Proof: From $a > c$ and $b > d$, we have $\frac{g_i}{f_i} < 1$ for all i according to (4). We further have:

$$1+\sum_{k=1}^{m-1}\prod_{i=1}^{k}\frac{g_i}{f_i}+\sum_{k=1}^{n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}+\left(\sum_{k=1}^{m-1}\prod_{i=1}^{k}\frac{g_i}{f_i}\right)\left(\sum_{k=1}^{n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}\right)\ge 1+\sum_{k=1}^{m+n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}$$

We thus obtain that:

$$\phi(\omega, m, n) = \frac{1}{1+\sum_{k=1}^{m-1}\prod_{i=1}^{k}\frac{g_i}{f_i}}\frac{1}{1+\sum_{k=1}^{n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}}$$

$$= \frac{1}{1+\sum_{k=1}^{m-1}\prod_{i=1}^{k}\frac{g_i}{f_i}+\sum_{k=1}^{n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}+\left(\sum_{k=1}^{m-1}\prod_{i=1}^{k}\frac{g_i}{f_i}\right)\left(\sum_{k=1}^{n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}\right)}$$

$$\le \frac{1}{1+\sum_{k=1}^{m+n-1}\prod_{i=1}^{k}\frac{g_i}{f_i}}$$

$$= \rho_A(\omega, m+n)$$

The result is therefore obtained and the proof is complete.

Remarks

Under $a > c$ and $b > d$, an interesting result is obtained. Namely, the hierarchy dynamic is more stable than unstructured dynamic if $a > c$ and $b > d$. This conclusion can be employed to effectively explain some situations in industrial economics.

Example 1

The industry with multiple poly monopolization firms is more stable than the others without monopolization. In an industry in which all firms with several monopolization groups, it is also considered as an s lection game on two hierarchy levels when a firm wants to enter this industry. In this game, if a > c and b > d, it is exceedingly difficult to invade this industry than those firms in free market by virtue of Theorem 2. Actually, a > c means that the profits with accept rejecting strategy are more than that with accepting strategy responding to rejecting strategy. b > d means that the profits with accept rejecting strategy are more than that with accepting strategy responding to accepting strategy. According to the aforementioned interesting conclusion in Theorem 2, an interesting phenomena in industrial economics is therefore rationally explained in the aforementioned example. We can similarly explain some other social and economic phenomena.

CONCLUDING REMARKS

In this paper, selection games in Nie (2007) are further analyzed and the fixation probability is obtained in hierarchy. We compare the structured games of two groups with the unstructured games, and an interesting result is obtained. Furthermore, some phenomena are rationally explained.

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