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Full Length Research Paper

Effects of Hall currents and slip condition on stable flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity

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Hall effects on the steady hydromagnetic flow due to non-coaxial rotations of a porous disk and a fluid at infinity with slip condition at the boundary has been studied. An exact solution of the governing equations has been obtained. The combined effects of Hall current, slip condition and suction or blowing are examined. It is found that both the primary velocity and the secondary velocity decrease with increase in Hall parameter. The heat transfer characteristic has also been studied on taking viscous and Joule dissipation into account. It is found that the critical Eckert number for which there is no flow of heat either from the disk to the fluid or from the fluid to the disk increases with increase in either Hall parameter or slip parameter.

Key words: Hall effects, slip condition, non-coaxial, heat transfer, porous disk and critical Eckert number.

INTRODUCTION

In an ionized gas where the density is low and the magnetic field is very strong, the conductivity will be a tensor. The conductivity normal to the magnetic field is reduced due to the free spiralling of ions and electrons about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both magnetic and electric fields. The phenomene well known in the literature, is called Hall effects. Due to the Hall current, a secondary motion ensues that the part of the flow takes place in the direction normal to both electric and magnetic fields. The study of magnetohydrodynamic flows with Hall currents has important engineering applications in the problem of magnetohydrodynamic generators and of Hall accelerators as well as in the flight magneto-hydrodynamic. The viscous incompressible flow due to non-coaxial rotations of a disk and a fluid at infinity has been considered by a number of researcher. Berker (1963) studied the viscous incompressive fluid between two parallel plates rotating non-coaxially with the same angular velocity. The flow due to a disk and a fluid at infinity which are rotating non-coaxially at slightly different angular velocities has been studied by Coirier (1972). Erdogan (1972, 1973) studied the same problem in the case of a porus disk, when the disk and the fluid at infinity rotates with the same and slightly different angular velocities respectively. The flow of a simple fluid in an orthogonal rheometer has been studied by Rajagopal (1992). Murthy and Ram (1978) have studied the MHD flow and heat transfer due to eccentric rotations of a porous disk and a fluid at infinity. Chakrabarti et al. (2005) considered the hydromagnetic flow due to a non-coaxial rotation of a porous disk and the fluid at infinity with same angular velocity. Hydromagnetic flow due to eccentrically non-conducting rotating porous disk and a fluid at infinity have been studied by Guria et al. (2007a). In all these studies, the effects of Hall current are neglected. Hall accelerations and in flight MHD. Hall effects on the viscous incompressible conducting fluid under various geometry have been considered by Sato (1961), Sherman and Sutton (1965), Pop and Soundalgekar (1974), Gupta (1975), Debnath et al. (1979), Datta and Jana (1975, 1977a, 1977b) and Jana et al. (1977). Recently, Guria et al. (2007b) have studied the Hall effects on the hydromagnetics flow due to non-

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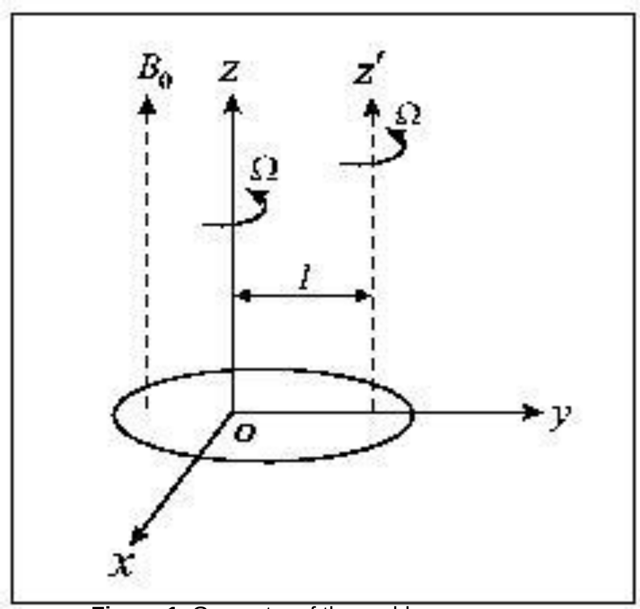

Figure 1. Geometry of the problem.

coaxial rotations of a porous disk and a fluid at infinity. Hall effects on the MHD flow generated by a rotating disk have been analyzed by Maleque and Sattar (2005b). Hayat et al. (2008) studied the Hall effects on unsteady flow due to non-coaxially rotation of a disk and a fluid at infinity.

The aim of the present study is to discuss the combined effects of Hall current and the slip condition at the disk on the steady conducting viscous incompressible flow due to non-coaxial rotations of a porous disk and a fluid at infinity. An exact solution of the governing equations has been obtained. It is found that the primary velocity decreases and the secondary velocity increases with increase in either slip-parameter λ or suction parameter S. The heat transfer characteristic of the problem has also been studied on taking viscous and Joule dissipation into account. It is found that the rate of heat transfer at the disk increases with increase in either Hall parameter m or slip-parameter λ. The non-dimensional force X^* exerted by the fluid on the disk decreases with increase in either slip parameter λ or Hall parameter m. On the other hand, the force Y^* exerted by the fluid on the disk increases with increase in Hall parameter m whereas it decreases with increase in slip parameter λ. Both the forces X^* and Y^* decrease with increase in suction parameter S.

METHODS

Mathematical formulation and its solution

Consider steady flow of a viscous incompressible conducting fluid occupying the space $z > 0$ and is bounded by an infinite porous non-conducting disk at $z = 0$. The axes of rotation of the disk and that of the fluid at infinity to be in the plane $x = 0$. The disk and the fluid at infinity rotate about z and z' -axes with the same uniform angular velocity Ω. The distance between the axes of rotation is l. A uniform magnetic field B_0 is applied perpendicular to the disk. The boundary conditions of the problem are:

$$u = -\Omega y + \beta\frac{du}{dz},\ v = \Omega x + \beta\frac{dv}{dz},\ w = -w_0 \text{ at } z = 0, \tag{1}$$

$$u = -\Omega(y - l),\ v = \Omega x,\ w = -w_0 \text{ as } z \to \infty, \tag{2}$$

where u, v, w are respectively the velocity components along x, y and z -directions and $w_0\ (> 0)$ is the suction velocity at the disk and β is the coefficient of a sliding friction.

The geometry of the problem (Figure 1) suggests that the velocity field in the flow is of the form:

$$u = -\Omega y + f(z),\ v = \Omega x + g(z),\ w = -w_0. \tag{3}$$

The generalized Ohm's law, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect (Cowling, 1957)

[redacted] (4)

B_0

where [redacted] is the current density vector, B [redacted] is the magnetic induction vector, [redacted] ctric field vector, ω_e is the cyclotron frequency and τ_e is the collision time of electron.

We shall assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is [redacted] ry small for the partially ionized gases. Assuming [redacted] $, B_y, B_z)$, the solenoidal relation $\nabla.$ [redacted] gives B_z = constant $= B_0$, everywhere in the flow. Furth [redacted] $= (j_x, j_y, j_z)$ be the components of the current [redacted], then the equation of the conservation of the charge $\nabla\cdot$ [redacted] gives j_z = constant. This constant is zero since $j_z = 0$ at the disk which is electrically non-conducting. Thus $j_z = 0$ everywhere in the flow. Again, for steady motion, the Maxwell's equation $\nabla\times$ [redacted] gives $\frac{\partial E_x}{\partial z} = 0$ and $\frac{\partial E_y}{\partial z} = 0$. This implies that E_x = constant and E_y = constant everywhere in the flow. In view of the above assumptions, Equation 4 gives:

$$j_x + m j_y = \sigma(E_x + vB_0), \tag{5}$$

$$j_y - m j_x = \sigma(E_y - uB_0), \tag{6}$$

where $m = \omega_e \tau_e$ is the Hall parameter.

At infinity, the magnetic field is uniform so that there is no current and hence, we have

$$j_x \to 0,\ j_y \to 0 \text{ as } z \to \infty. \tag{7}$$

On the use of Equation 7, Equations 5 and 6 yield

$$E_x = -\Omega B_0 x,\ E_y = -\Omega B_0 (y - l), \tag{8}$$

everywhere in the flow.

Substituting the above values of E_x and E_y in the equations (5) and (6) and solving for j_x and j_y, we get:

$$j_x = \frac{\sigma B_0}{1+m^2}[g - m(\Omega l - f)], \tag{9}$$

$$j_y = \frac{\sigma B_0}{1+m^2}[(\Omega l - f) + mg]. \tag{10}$$

Substituting Equation 3 and using Equations 9 and 10, the Navier-Stokes equations along x and y directions become:

$$-w_0 \frac{df}{dz} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \Omega^2 x + \nu \frac{d^2 f}{dz^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}\left[(\Omega l - f) + mg\right] + \Omega g, \tag{11}$$

$$-w_0 \frac{dg}{dz} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \Omega^2 y + \nu \frac{d^2 g}{dz^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[g - m(\Omega l - f)\right] - \Omega f. \tag{12}$$

The boundary conditions for $f(\eta)$ and $g(\eta)$ are

$$f(0) = \beta \frac{df(0)}{dz},\ g(0) = \beta \frac{dg(0)}{dz}, \tag{13}$$

$$f(\infty) = \Omega l,\ g(\infty) = 0. \tag{14}$$

On the use of infinity condition (14) (Erdogan, 1977), Equations 11 and 12 yield

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \Omega^2 x, \tag{15}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \Omega^2 y. \tag{16}$$

Using Equations 15 and 16, Equations 11 and 12 become

$$-w_0 \frac{df}{dz} - \Omega g = \nu \frac{d^2 f}{dz^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}\left[(\Omega l - f) + mg\right], \tag{17}$$

$$-w_0 \frac{dg}{dz} - \Omega(\Omega l - f) = \nu \frac{d^2 g}{dz^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[g - m(\Omega l - f)\right]. \tag{18}$$

Introducing

$$\eta = \sqrt{\frac{\Omega}{\nu}}\, z,\ M^2 = \frac{\sigma B_0^2}{\rho \Omega},\ S = \frac{w_0}{\sqrt{\Omega \nu}} \tag{19}$$

and combining Equations 17 and 18, we have

$$\frac{d^2 F}{d\eta^2} + S\frac{dF}{d\eta} - \left[\frac{M^2}{1+m^2} + i\left(1 + \frac{mM^2}{1+m^2}\right)\right]F = 0, \tag{20}$$

where

$$F(\eta) = 1 - \frac{f+g}{\Omega l}. \tag{21}$$

The corresponding boundary conditions for $F(\eta)$ are

$$F(0) = 1 + \lambda \frac{dF(0)}{d\eta} \text{ and } F(\infty) = 0, \tag{22}$$

where λ is a slip parameter.

The solution of Equation 20 subject to the boundary conditions (22) is

$$F(\eta) = \frac{\exp\left[-\left(\frac{S}{2} + \alpha + i\beta\right)\eta\right]}{1 + \lambda\left(\frac{S}{2} + \alpha + i\beta\right)}, \tag{23}$$

where

$$\alpha, \beta = \frac{1}{2\sqrt{2}}\left[\left\{\left(S^2 + \frac{4M^2}{1+m^2}\right)^2 + 16\left(1 + \frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}} \pm \left(S^2 + \frac{4M^2}{1+m^2}\right)\right]^{\frac{1}{2}}. \tag{24}$$

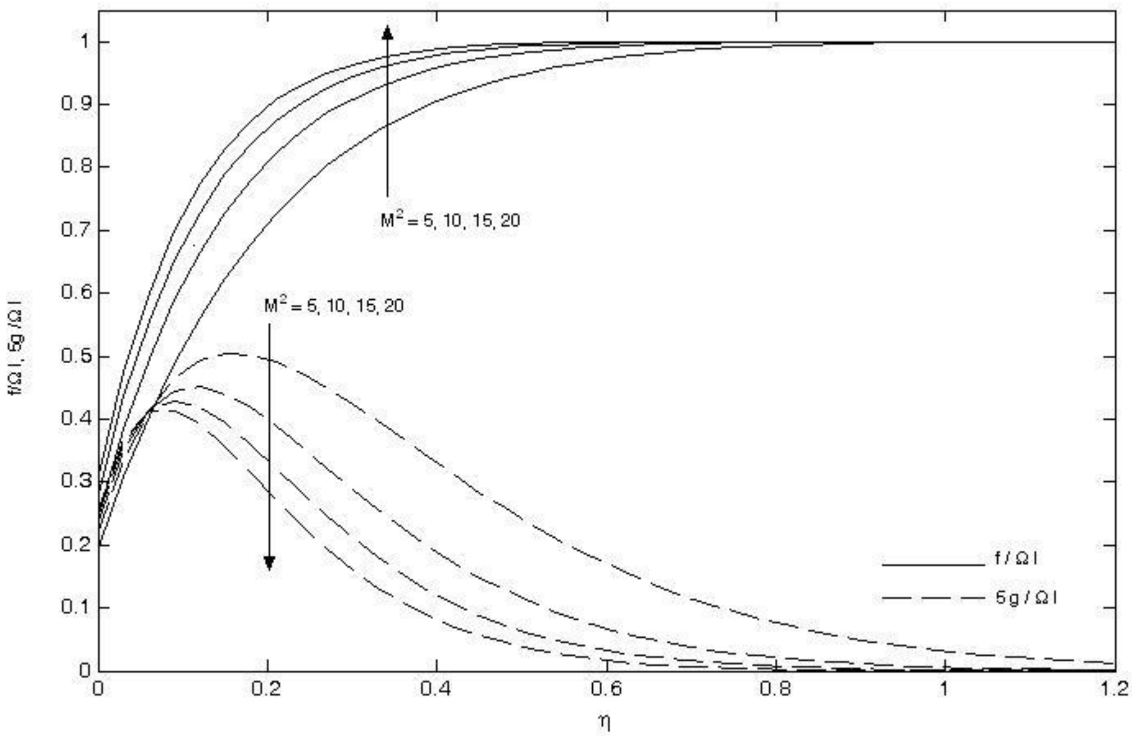

Figure 2. Variations of $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for $S = 1$, $m = 0.5$ and $\lambda = 0.05$.

Using Equation 21 and on separating into a real and imaginary parts, Equation 21 yields

$$\frac{f}{\Omega l} = \frac{e^{-\left(\frac{S}{2}+\alpha\eta\right)}}{\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2} \times \left[\left\{1+\lambda\left(\frac{S}{2}+\alpha\right)\right\}\cos\beta\eta-\lambda\beta\sin\beta\eta\right], \quad (25)$$

$$\frac{g}{\Omega l} = \frac{e^{-\left(\frac{S}{2}+\alpha\eta\right)}}{\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2} \times \left[\left\{1+\lambda\left(\frac{S}{2}+\alpha\right)\right\}\sin\beta\eta+\lambda\beta\cos\beta\eta\right]. \quad (26)$$

The solutions given by Equations 25 and 26 are valid for both the suction ($S > 0$) and the blowing ($S < 0$) at the disk. If $\lambda = 0$ and $m = 0$ then the equations (25) and (26) coincide with the Equation 5 of Murthy and Ram (1978). Further if $\lambda = 0$, $S = 0$ and $M^2 = 0$, then Equation 21 is reduced to Equation 13 of Erdogan (1976).

RESULTS AND DISCUSSION

Equations 25 and 26 show that there exists a single-deck boundary layer near the disk and the thickness of this layer is of the order of $O\left\{\left(\frac{S}{2}+\alpha\right)^{-1}\right\}$. The thickness of this boundary layet decreases with an increase in either suction parameter S or magnetic parameter M^2 because α , as shown in the Equation (24), decreases with an increase in either suction parameter S or magnetic parameter M^2 . On the other hand, the boundary layer thickness increases with an increase in Hall parameter m as α decreses with an increse in Hall parameter m . To study the combined effects of Hall current and slip condition on the steady flow of a conducting viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity, the dimensionless velocity components $f/\Omega l$ and $f/\Omega l$ are plotted against in Figures 2 to 5 for several values of Hall parameter m , slip parameter λ and suction parameter S . Figure 2 shows that the primary velocity $f/\Omega l$ increases while the secondary velocity $g/\Omega l$ decreases with an increase of magnetic parameter M^2 . It is observed from Figure 3 that the primary velocity $f/\Omega l$

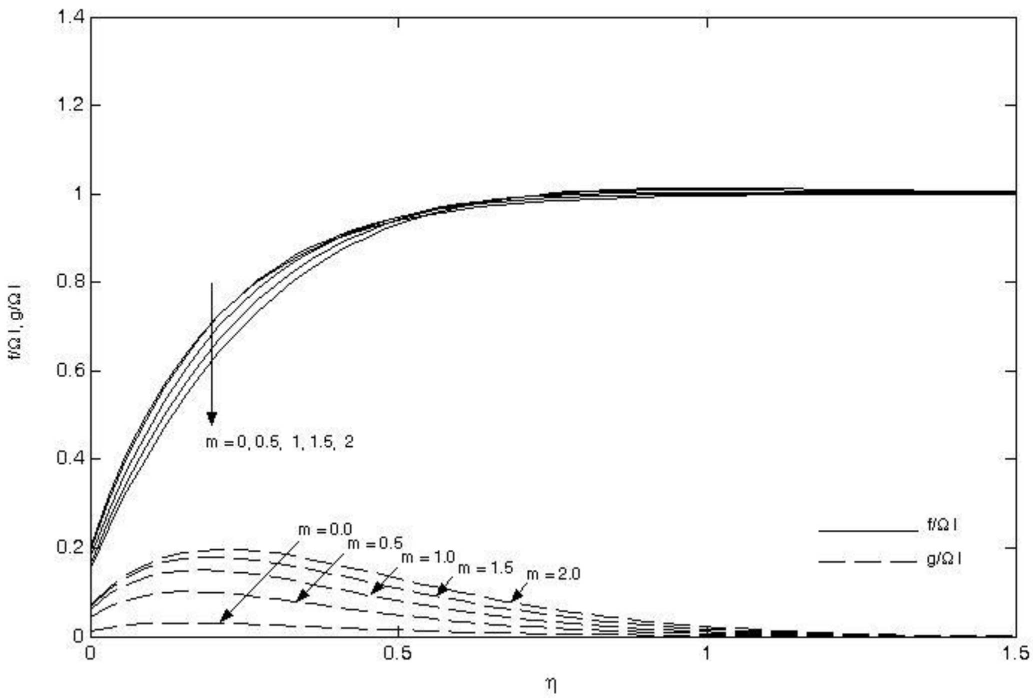

Figure 3. Variations of $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for $M^2 = 5$, $S = 1$ and $\lambda = 0.05$.

decreases whereas the secondary velocity $g / \Omega l$ increases with an increase of Hall parameter m. It is seen from Figure 4 that the primary velocity $f / \Omega l$ increases with an increase in slip parameter λ. On the other hand, the secondary velocity $g / \Omega l$ increases near the disk while it decreases away from the disk with an increase in λ. Figure 5 reveals that the primary velocity $f / \Omega l$ increases while the secondary velocity $g / \Omega l$ decreases with an increase in suction parameter S. The components of the force exerted by the fluid on the disk along the x - and y -directions are:

$$\theta(\eta) = \begin{cases} 1 - e^{-S Pr \eta} + \dfrac{Pr\, Ec \left\{ \left(\frac{S}{2} + \alpha \right)^2 + \beta^2 + \frac{M^2}{1+m^2} \right\}}{\left\{ \left(1 + \frac{S\lambda}{2} + \lambda\alpha \right)^2 + \lambda^2 \beta^2 \right\} (S + 2\alpha)(S + 2\alpha - SPr)} \times \left[e^{-(S+2\alpha)\eta} - e^{-S Pr \eta} \right] & \text{for } S\, Pr \neq S + 2\alpha, \\ 1 - e^{-SPr\eta} + \dfrac{PrEc \left\{ \left(\frac{S}{2} + \alpha \right)^2 + \beta^2 + \frac{M^2}{1+m^2} \right\}}{\left\{ \left(1 + \frac{S\lambda}{2} + \lambda\alpha \right)^2 + \lambda^2 \beta^2 \right\} SPr} \times \eta e^{-S Pr \eta} & \text{for } S\, Pr = S + 2\alpha \end{cases} \tag{27}$$

where Σ denotes the surface of the disk of radius r_0, and $\tau_{xz}(0)$ and $\tau_{yz}(0)$ are the shear stresses on the disk given by:

$$\tau_{xz}(0) = \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)_{z=0}$$

$$\tau_{yz}(0) = \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)_{z=0}. \tag{28}$$

Using Equations 3 and 28, Equation 27 becomes:

$$X = \pi \mu r_0^2 f'(0) \text{ and } Y = \pi \mu r_0^2 g'(0), \tag{29}$$

where $f'(0)$ and $g'(0)$ are obtained from Equations 25 and 26.

In the case of suction at the disk, Equations 27 give, on using Equations 23 and 24

$$X^* = \frac{X}{\pi \mu r_0^2 \left(\frac{\Omega}{\nu} \right)^{\frac{1}{2}} \Omega l}$$

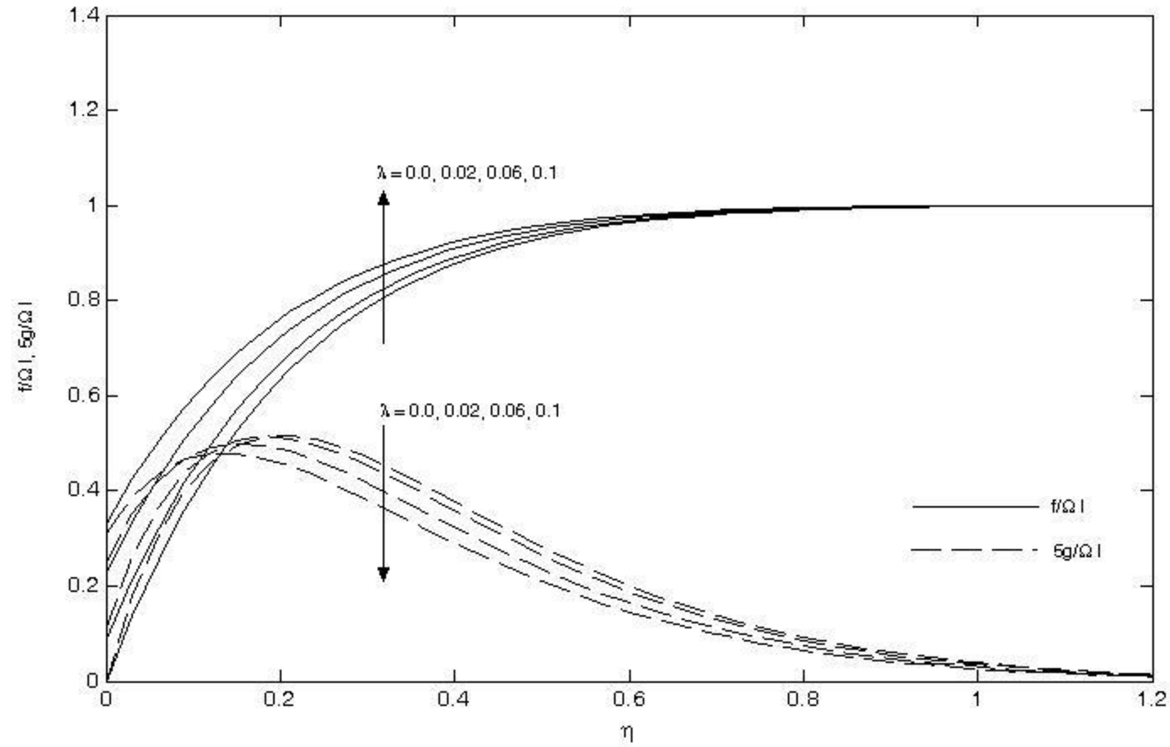

Figure 4. Variations of $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for $M^2 = 5, S = 1$ and $m = 0.5$.

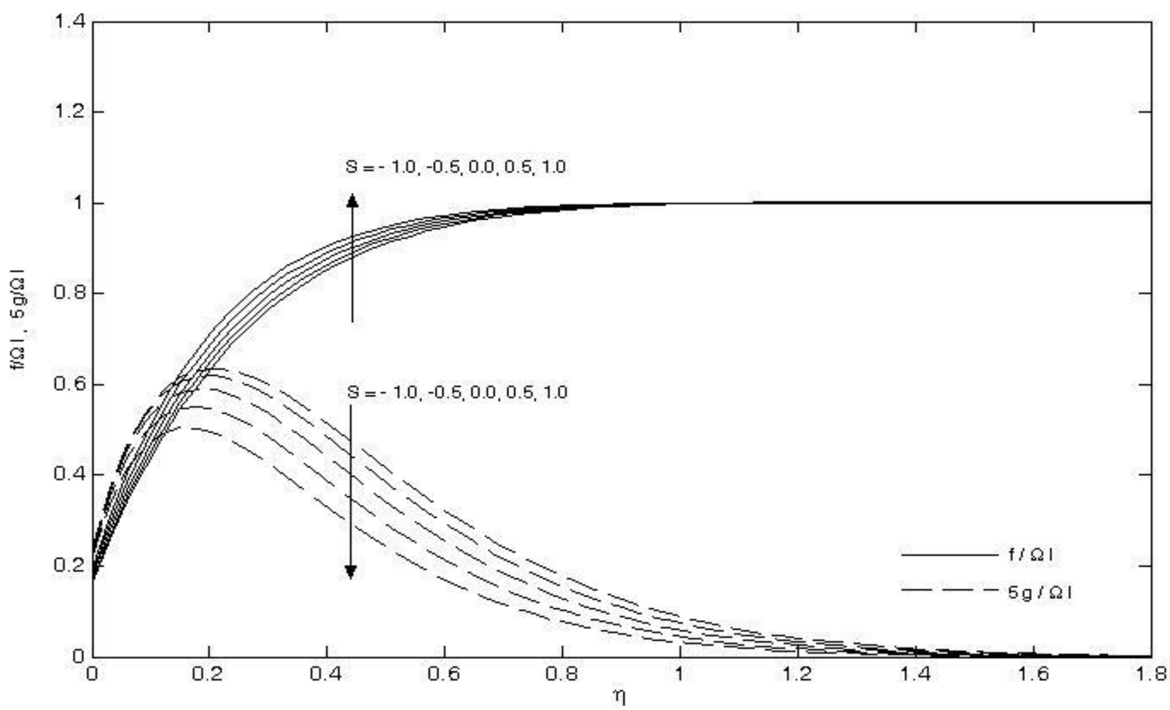

Figure 5. Variations of $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for $M^2 = 5, m = 0.5$ and $\lambda = 0.05$.

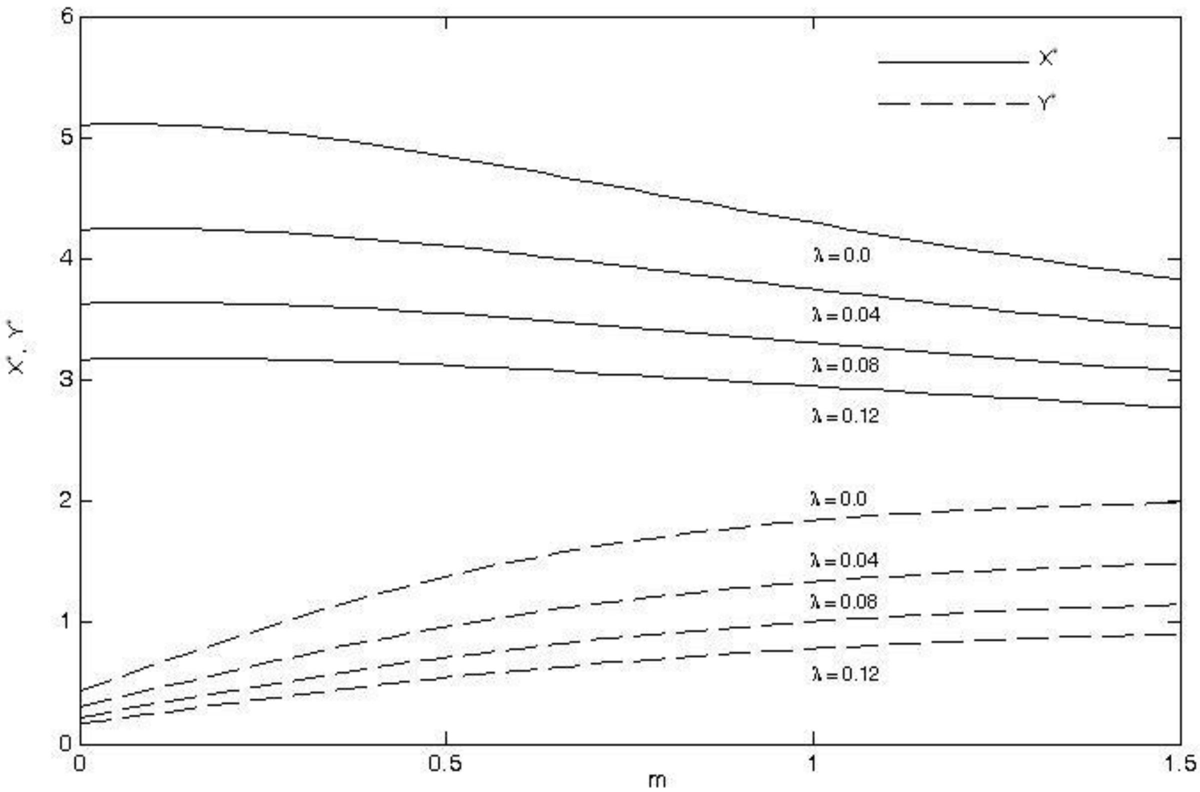

Figure 6. Variations of X^* and Y^* for $M^2 = 5, S = 1.$

$$= \frac{\left(\frac{S}{2}+\alpha\right)\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)+\lambda\beta^2}{\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2}\ .$$

$$Y^* = \frac{Y}{\pi\mu r_0^2\left(\frac{\Omega}{\nu}\right)^{\frac{1}{2}}\Omega l}$$

$$= \frac{\beta\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)-\lambda\beta\left(\frac{S}{2}+\alpha\right)}{\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2}\ . \qquad (30)$$

Similarly, in the case of blowing at the disk, the corresponding forces are obtained by replacing

$S = -S_1$ $(S_1 > 0)$ in Equation 30.

The numerical results of the non-dimensional forces X^* and Y^* on the disk ($\eta = 0$) are shown in Figures 6 and 7 against m for different values of slip parameter λ with $M^2 = 5$ and $S = 1$. It is observed from Figure 6 that the dimensionless force X^* decreases with increase in either λ or m . On the other hand, it is observed that the force Y^* increases with increase in m when λ is fixed, while for fixed values of m , it decreases with increase in λ . It is seen from Figure 7 that both the forces X^* and Y^* decrease with increase in suction parameter S . It is well known that suction causes reduction in the forces exerted by the fluid on the disk.

The torque exerted by the fluid on the disk is given by

$$M^* = \int_{\Sigma} [\, x\tau_{yz}(0) - y\tau_{xz}(0)]\, dS. \qquad (31)$$

Using Equations 28 and 29 in Equation 31, we get $M^* = 0$. It means that the non-coaxial rotation of the disk and the fluid at infinity has no influence on the torque.

Heat transfer

We shall now determine the fluid temperature distribution T and rate of heat transfer for the case of suction at the disk. The energy equation can be written for the problem under consideration is:

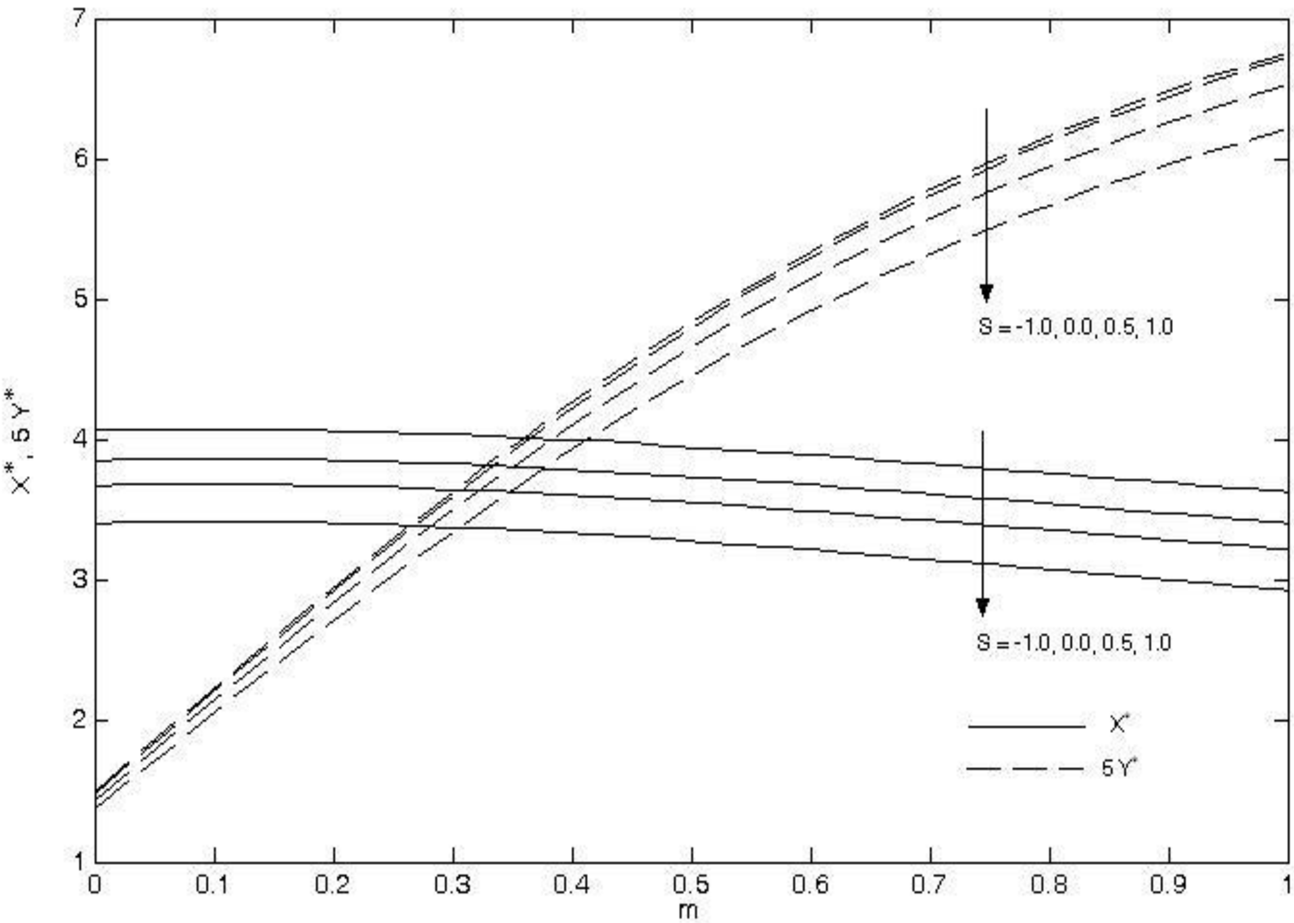

Figure 7. Variations of X^* and Y^* for $M^2 = 5$ and $\lambda = 0.05$.

$$-\rho c_p w_0 \frac{dT}{dz} = k\frac{d^2T}{dz^2} + \mu\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] + \frac{1}{\sigma}\left(j_x^2 + j_y^2\right), \quad (32)$$

where c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid and the last two terms on the right hand side of (32) represent the viscous dissipation and the Joule dissipation respectively. The temperature boundary conditions are:

$$T = T_w \text{ at } z = 0 \text{ and } T \to T_\infty \text{ as } z \to \infty, \quad (33)$$

where T_w is the constant temperature of the disk and T_∞ is the uniform temperature of the ambient fluid where we assume that $T_w > T_\infty$.

Introducing:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Ec = \frac{\Omega^2 l^2}{c_p\,(T_w - T_\infty)},\ Pr = \frac{\mu c_p}{k}, \quad (34)$$

and using Equations 3, 19, 25 and 26, Equation 32 become

$$\frac{d^2\theta}{d\eta^2} + SPr\frac{d\theta}{d\eta} = -\frac{PrEc\left\{\left(\frac{S\lambda}{2}+\alpha\right)^2 + \beta^2 + \frac{M^2}{1+m^2}\right\}}{\left(1+\frac{S\lambda}{2}+\lambda\alpha\right)^2 + \lambda^2\beta^2}\, e^{-(S+2\alpha)\eta}. \quad (35)$$

The corresponding boundary conditions for $\theta(\eta)$ are

$$\theta(0) = 1 \text{ and } \theta(\infty) = 0. \quad (36)$$

The solution of Equation 35 subject to the boundary conditions (36) is:

$$\theta(\eta)=\begin{cases}1-e^{-S\Pr\eta}+\dfrac{\Pr Ec\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}(S+2\alpha)(S+2\alpha-S\Pr)}\times\left[e^{-(S+2\alpha)\eta}-e^{-S\Pr\eta}\right] & \text{for } S\Pr\neq 2\alpha,\\ \dfrac{\Pr Ec\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}S\Pr}\times\eta e^{-S\Pr\eta} & \text{for } S\Pr=2\alpha\end{cases} \quad (37)$$

If $\lambda=0$ and $m=0$, then the above Equation 37 coincides with Equations 17 and 18 of Murthy and Ram (1978). The non-dimensional rate of heat transfer q_w at the disk $\eta=0$ is given by:

$$q_w=-k\left(\frac{dT}{dz}\right)_{z=0}=-\frac{k\Omega l}{\nu(T_w-T_\infty)}\left(\frac{d\theta}{d\eta}\right)_{\eta=0}. \quad (38)$$

Equation 37 gives:

$$-\left(\frac{d\theta}{d\eta}\right)_{\eta=0}=S\Pr-\frac{\Pr Ec\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}(S+2\alpha)} \quad (39)$$

Since $T_w>T_\infty$, it follows from Equations 38 and 39 that heat will flow from the disk to the fluid if:

$$Ec<\frac{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}(S+2\alpha)S}{\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}. \quad (40)$$

On the other hand, heat will flow from the fluid to the disk if:

Table 1. Critical Ekart number Ec^* for $M^2=5.0$ and $S=1$.

$\lambda \backslash m$	0.0	0.5	1.0	1.5
0.00	0.32680	0.32972	0.35278	0.38007
0.04	0.47393	0.47099	0.48633	0.50789
0.08	0.64850	0.63908	0.64455	0.65836
0.12	0.85049	0.83397	0.82743	0.83147

$$Ec>\frac{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}(S+2\alpha)S}{\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}. \quad (41)$$

It is clear from Equation 39 that there will be no flow of heat either from or towards the disk when

$$Ec^*=\frac{\left\{\left(1+\dfrac{S\lambda}{2}+\lambda\alpha\right)^2+\lambda^2\beta^2\right\}(S+2\alpha)S}{\left\{\left(\frac{S}{2}+\alpha\right)^2+\beta^2+\dfrac{M^2}{1+m^2}\right\}}. \quad (42)$$

The values of the critical Eckert number Ec^* are given in Table 1. It is observed that the critical Eckert number increases with increase in slip parameter λ while it first decreases (except $\lambda=0$) reaches a minimum and then increases with increase in m . The inequality (41) shows that heat may flow from the fluid to the disk even if the temperature of the disk is greater than that of the free-stream temperature $T_w>T_\infty$. The reversal of heat flow can be explained on physical ground. It is seen that if there is significant viscous dissipation near the disk then the temperature of the fluid near the disk may exceed the disk temperature. This will cause flow of heat from the fluid to the disk even though $T_w>T_\infty$. It is interesting to note from Equation 37 that the thermal boundary layer has a double deck structure for $SPr\neq S+2\alpha$. The thickness of these layers are $O\left(\frac{1}{SPr}\right)$ and $O\left(\frac{1}{S+2\alpha}\right)$. On the other hand, for $SPr=S+2\alpha$ there is a single-deck thermal boundary layer with thickness of order of $O\left(\frac{1}{S+2\alpha}\right)$. It is seen from (22) that the layer with thickness of order of $O\left(\frac{1}{S+2\alpha}\right)$ decreases with increase in either S or m .

The values of the rate of heat transfer $-\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$ are given in Table 2 for different values of λ and m . It is

Table 2. Rate of heat transfer $-10\times\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$ for $Pr = 0.025$, $M^2 = 5$, $S = 1$ and $Ec = 0.3$.

λ/m	0.0	0.5	1.0	1.5
0.00	0.00205	0.00225	0.00374	0.00527
0.04	0.00918	0.00908	0.00958	0.01023
0.08	0.01343	0.01326	0.01336	0.01361
0.12	0.01618	0.01601	0.01594	0.01598

clear from the Table 2 that the rate of heat transfer at the disk increases with increase in slip-parameter λ. On the other hand, the rate of heat transfer first decreases (except $\lambda = 0$) reaches a minimum and then increases with increase in m.

Conclusion

The effects of both Hall current and slip condition on the steady hydromagnetic flow of a viscus incompressible conducting fluid due to non-coaxial rotations of a porous disk and a fluid at infinity is studied. An exact solution of the governing equations has been obtained. It is found that both the primary velocity and the secondary velocity decrease with increase in Hall parameter. It is also found that the primary velocity decreases and the secondary velocity increases with increase in either slip-parameter λ or suction parameter S. It is observed that the critical Eckert number for which there is no flow of heat either from the disk to the fluid or from fluid to the disk increases with increase in either Hall parameter m or slip parameter λ. Further, it is observed that the rate of heat transfer at the disk increases with increase in slip-parameter λ. It is interesting to note that the non-coaxial rotations of a porous disk and fluid at infinity has no influence on the torque exerted by the fluid on the disk. S. The suction at the disk causes reduction in the forces exerted by the fluid on the disk.

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