African Journal of Physics Vol. 2 (2), 7 pages, February, 2014. Available online at www.internationalscholarsjournals.org © International Scholars Journals

Full Length Research Paper

A study of potential physics instructor on mathematical models in physics

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Accepted 15 December, 2013

The aims of this study are "to reveal potential physics teachers' characterization of the relationship between mathematics and physics" and "to analyze their abilities to model physical phenomena". For these purposes, an achievement test consisting of two parts has been administered to 24 student teachers of physics. The first part aims to reveal their opinions about the relationship between mathematics and physics and the second part aims to determine their abilities about the construction of a model. The analysis shows that the candidate physics teachers state the role of mathematics in physics in three different ways and that although they generate their own methods for modeling physical phenomena they have serious difficulties in forming a model. As a recommendation of this study, it can be said that model and modeling process should be tackled as an object and model teaching should not content with model as a tool.

Key words: Mathematics, physics, modelling, student's teachers.

INTRODUCTION

Mathematics and physics play an imminent role for each other. Indeed there is a reciprocal relationship between mathematics and physics which goes back to centuries ago and this "relationship has been especially strong since Galileo established the modern conception of the scientific method, wherein mathematics plays a crucial role, in the first decades of the 17th century" (Helfgott, 2004). In line with this, Helfgott (2004) notes that there are many notable mathematicians who have contributed to physics: Descartes, Fermat, Leibniz, the Bernoulli brothers, Euler, D'Alembert, Lagrange, Laplace, Cauchy, Gauss Riemann, Poincaré and so on.

Roughly speaking, mathematics allow describing physical phenomena and physics constitute an application field for mathematics. Therefore, this relationship can be characterized in two ways: on the one hand mathematicians use physical concepts and arguments and on the other hand physicians use mathematical concepts and methods. However, the relationship between these two disciplines cannot be reduced to "physics are a domain of application of mathematics" and "mathematics is the language of physics" (Tzanakis, 2002). Therefore, to characterize the relationship between these disciplines, we need a term more large than "language and domain of application" (Chaachoua and Saglam, 2006). Blum (2002) proposes the term 'applications and modeling' to describe any relation between real word and mathematics. These terms perfectly stand for the relation between mathematics and physics science. Principally, when solving real word's problems the physicists try to obtain a mathematical model that describes or represents some aspect of the real situation (Berry and Houston, 1995). In line with this information, it can be said that the model concept is crucial for physical sciences.

Model: Tool or object?

Douady (1986) indicates two different status (tool and object) of a concept or knowledge and she mentions tool-object dialect for concepts. A mathematical concept is called tool when it is used in order to solve a problem. On the other hand, a concept is considered as an object when somebody focuses on its definition, properties and so on. When we apply this dialect for the model concept, we can say that models are sometimes tool and sometimes object (Rodriguez, 2007) . Thus, analysis of the current literature on model and modeling with this point of view shows that there have been two main axes in the literature: while some studies focus on model

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instruction itself others studies focus on forming models (Rodriguez, 2007; Garcia et al., 2006). There have been many studies related to the former axis sampling students' skills of using models (Gobert and Buckley, 2000; Cullin and Crawford, 2002; Mevarech and Kramarski, 2004). Considering the importance of forming models for physics problems, we adopted the latter axis; object status of the model for the present study.

Purpose of the study

Based on the information given above, the relationship between mathematics and physics, and model construction process will be investigated in the present study. In line with this, we worked out our research purposes as following:

(i) To reveal the student teachers' characterization of the relationship between mathematics and physics,
(ii) To analyze the student teachers' abilities to model physical phenomena by using mathematics.

METHODS

Subject

Our subjects were 24 student teachers of physics department from the Faculty of Education of a Turkish University. For the selection, two criteria were sought: having taken the courses presenting mathematical issues used in physics - Differential Equations, Mathematical methods in Physics – and succeeding in them academically. The contents of these courses are introduced below.

The lecture entitled "Differential Equations" aims to introduce the relationships between real-life situations and differential equations. A sourcebook (Edwards and Penny, 2005) is mainly used in this lecture which concerns generally the following subjects: First-order differential equations with applications, Linear Differential equations of second and higher order, and systems of differential equations. The book makes use of Maple or Mathematical software for symbolic computation and MATLAB for numeric computation.

The lecture, "Mathematical methods in Physics", focuses on mathematical concepts most often used in physics and shows their applications. This lecture contains the following subjects*:* Functions of complex variables, Analytic functions, Complex integrations and Cauchy theorem, Taylor and Laurent series, Residues, Conform transformations, Schwartdz-Cristoffel transformation. Two books were recommended as sourcebook for the course: Mathematical Methods for Physicists (Arfken, 1985) and Mathematical methods in Physics and Engineering (Karao lu, 1994). Examining the first book, it was determined that the units associated with the content of the course were covered in three phases; the relevance of the issue with physics, basic information and explanations related to the issue, and sample applications. It was determined that this book mainly aims to demonstrate students to use mathematics, generally as a model, in physics by particularly including physics problems. When the second sourcebook investigated, it can be seen that basic and advanced mathematical knowledge in this book was handled entirely in a practical point of view. Besides, this book also aims to teach how to apply mathematical results in daily life handily.

The analysis above states that mathematics issues are instructed in basic and advanced level and their applications in physics are presented in "Differential Equations" and "Mathematical methods in Physics" courses. Briefly, these courses focus on mathematics that is used in physics. In the books, it was well examined which physics units related to mathematics units and these subjects were used in sample physics problems, however; modeling process was not considered properly. Indeed modeling has a few steps: transition from real situation to real world model, transition from real world model to mathematical model, obtaining mathematical results by using mathematical model, and returning to the real situation (Kaiser, 2005; Rodriguez, 2007), and actually the contents of the sourcebooks did not comply with these steps.

Data collection

It is well known that modeling is performed by means of various mathematical concepts. Considering the richness and the complexity of the modeling process, this study focuses on modeling by differential equations. In this study, an achievement test, developed by the researchers, was used as data collection tool. The test consists of two parts. The first part consists of three questions. Each question aims to reveal different aspects of the students' opinions about the relation between mathematics and physics. Questions of this group are as follows: What is the place of mathematics in physics? What is the role of mathematics in physics? And how do you define a mathematical model?

The second part of the test aims to reveal the abilities of the student teachers about the construction of a model. Based on the literature, Saglam (2004) noted that, there were two approaches to construct a mathematical model: "experimental approach and theoretical approach". While the experimental approach is based on data of an experiment –frequently on an experimental curve-, theoretical approach is based on a law obtained by a physical model (Guillon, 1995). Considering this fact, we proposed in this study three open-ended questions. While Question 1 and Question 3 are about theoretical approach, the second one which based on "inverse tangent" problem asked by DeBeaune in 1638 is related to experimental approach. We introduce these questions and their solutions below:

Question 1 and its solution

Consider an object thrown up at a vertical velocity V_0. Establish the differential equation representing the movement of the object at a moment t. Find the expressions for height and speed.

To solve this question, student teachers must use the theoretical approach which makes reference inevitably to the second law of Newton. The solution is as follows:

F = m.a $\quad m.\frac{dv}{dt} = -m.g$ et $y''(t) = -g(t)$

y(0) = 0 , v(0) = v_0

$$ {}^{t}_{0}\; y''(t).dt = - \; g.dt $$

By completing this stage, they can find the following expressions:

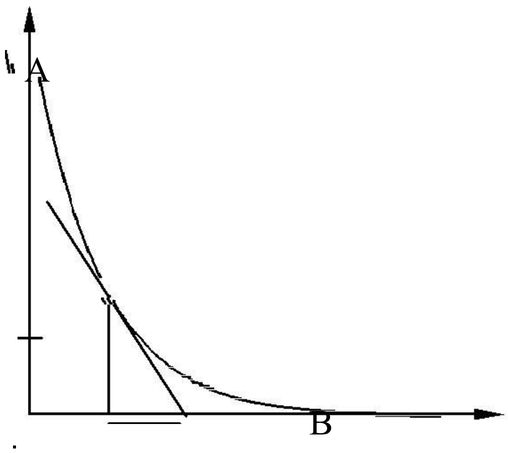
Heights' expression: $y(t) = -\frac{1}{2}.g.t^2 + v_0 .t$ and speeds' expression: $v(t) = v_0 - g.t$

Question 2 and its solution

The Figure below shows the curve AB, which describes the evolution of tension "u_R" according to time "t", observed on the screen of an oscilloscope branched at the points (input/output terminal) of a resistance.

(a) Establish the differential equation describing the curve AB in which “a” is constant on every point. (Note that an equation of the tangent line to a curve at one point x^0 is: $y'(x^0) = \frac{y - y_0)}{x - x_0}$

(a) Find the nature of the solution to this differential equation?



This problem draws one’s inspiration from the problem named “inverse tangent” –proposed by De Beaune (Saglam, 2004) – leads the student teachers to use the experimental approach for constructing the mathematical model. In fact, this problem requires working on the equation of the tangent line initially drawn on the curve proposed to reach the model. The solution of this problem is:

(a) By replacing the values observed on the graph in the equation of tangent line, the following relation is reached: $y'(x_0) + \frac{1}{a}.y_0 = 0$.

As *a* is constant and on the graph *y(x)* shows *u(t)*, the following relation can be deduced: $u'(t) + \frac{1}{a}.u(t) = 0$.

(b) The solution to the differential equation $y'(x) + ay(x) = 0$ is: $x \quad Ce^{-ax}$ $(C \in \Re)$ from which the solution to the differential equation ($u'(t) + \frac{1}{a}.u(t) = 0$) is $t \quad Ce^{-\frac{1}{a}t}$.

Question 3 and its solution

Consider the circuit below, including a resistance R, a coil inductor L and its resistance intern r. Initially, the current of the coil is equal to i_0.

(a) On the basis of the electrokinetic’s laws, establish the differential equation representing the following circuit RL at any time t?

(b) Find the evolution of the current passing through the coil?

This question, considered as a usual task for students, requires establishing the differential equation of the electrical circuit *RL* and finding a family of solutions to this differential equation. This could be done in the following way:

(a) By using the Kirchhoff’s voltage law ($\sum_{\substack{Closed\\loop}} \Delta V = 0$), which means that the sum of the voltage (potential) in any loop must equal zero, we can describe the relationship: $u_R + u_L = 0$.

As $u_R = i(t).R$ and $u_L = L.\frac{di(t)}{dt} + r.i(t)$, the differential equation representing this loop is: $L.\frac{di(t)}{dt} + (R + r).i(t) = 0$.

(b) By referring to the algebraic methods (xCe^{-ax} $(C \in \Re)$), the general solution is found as the following form: $i(t) = Ce^{-\frac{R+r}{L}.t}$ $(C \in \Re)$

As $i(t = 0) = i_0$, $C = i_0$ and $i(t) = i_0.e^{-\frac{R+r}{L}.t}$.

Data analysis

In this study, different analysis methods have been applied with regard to the attributions of different data types:

(i) To uncover students’ characterization of the relationship between mathematics and physics the qualitative data obtained were analyzed via phenomenographic analysis. This method employs definition of qualitative data sets to determine how the same phenomenon is perceived by different individuals (Marton and Yang Pong, 2005). Accordingly, firstly; preliminary data categories were formed by detecting the similarities among the answers given for the questions in the first part of the data gathering tool and then the main categories were formed after the data reviewed twice.

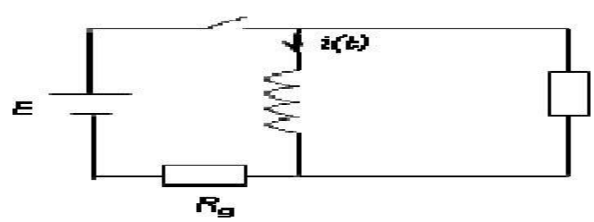
(ii) To analyze the students’ ability to construct a model based upon students’ solutions to the questions administered, the praxeological method, introduced by Chevallard (1998), was used. This analysis method helps to determine the techniques an individual uses to complete a task. By using this analysis method, the students’ different thinking styles –techniques- they use during the construction of models, were determined as a result of the examination of the answers given for the questions administered.

RESULTS

The data are presented in terms of the purposes of this study.

Students’ characterization of the relationship between mathematics and physics

Figure 1 maps students’ descriptions of the relationship between mathematics and physics. 10 description categories were defined under 3 main titles. Table 1 shows the expressions the students used to define the



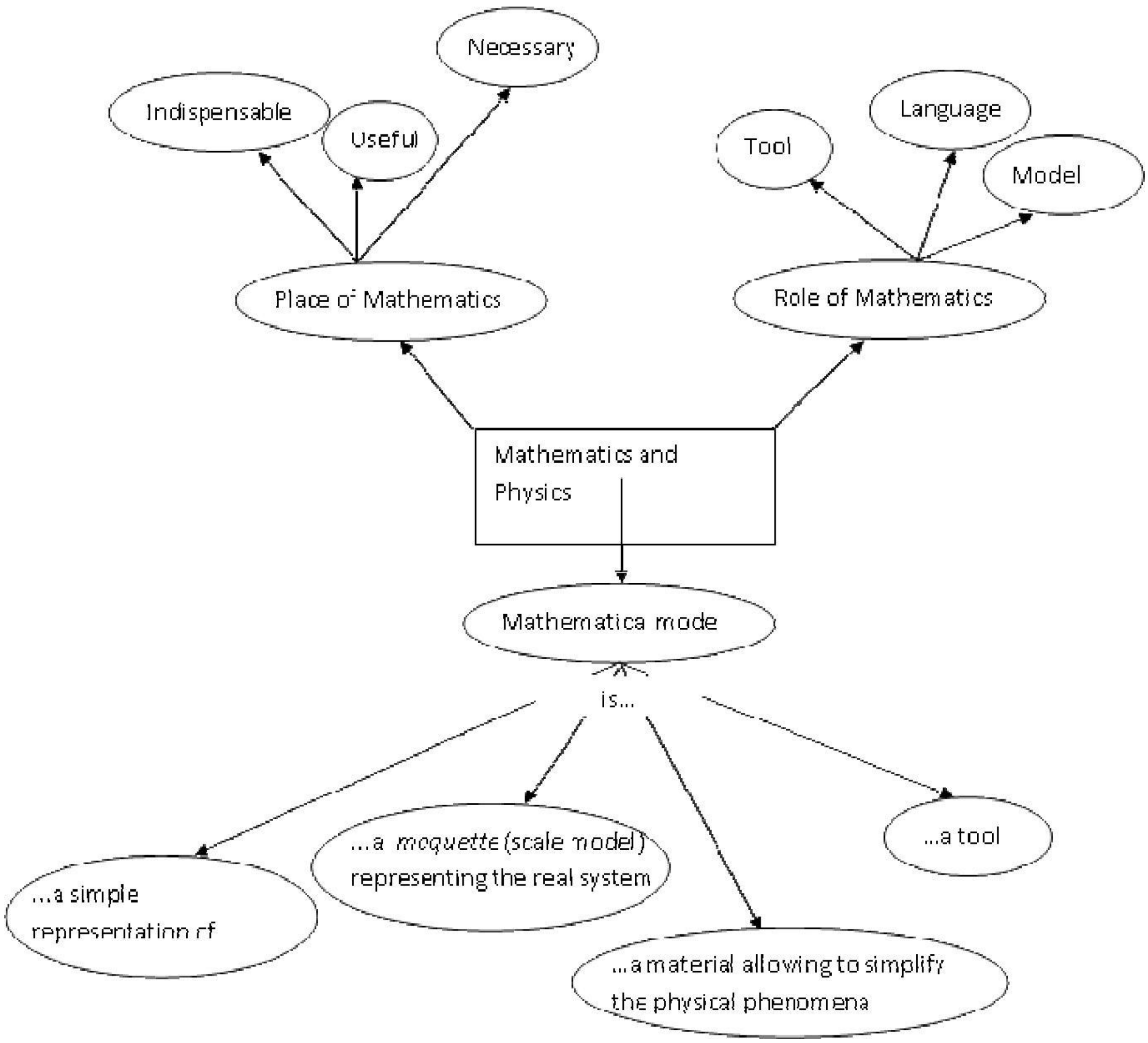


Figure 1. Students' descriptions of relationship between Mathematics and Physics.

relation between mathematics and physics that were given with the repetition frequencies. The students preferred to use the expressions; "indispensable, necessary and useful" to state the significance of mathematics in physics (Table 1). When we check the frequencies, it was observed that considerable number of the students said that mathematics is indispensable and necessary for physics (42% and 29%, respectively). The importance of mathematics in physics, the ratio of using mathematics in physics' curriculum and time-span saved for mathematical calculations in physics courses may have oriented them to answer in that way. The rest of them, on the other hand, argued that mathematics is useful for physics (25%). A belief stating that physics courses should be carried out in experimental environments, may have made these students grow the idea of physics and can do without mathematics.

The analysis of the answers for the question asking for the definition of the role of mathematics in physics shows that there are three different roles in the students' mind: tool, language and model (Table 1). The role most often cited (53%) by student teachers is "tool". This first role is followed by language (17%), and model (11%). The students' referring mathematics as a tool, instead of using mathematics scientifically in physics as a language or model that may be attributed to those applications in the curriculum.

The analysis of the answers for the last question in the first part of the data collecting tool related to the role of mathematics in physics shows that mathematical models are defined differently by students as: a simple representation of physical phenomena, a maquette (scale model) representing the real system, a material allowing to simplify the physical phenomena (29, 29 and 21%, respectively.) Even though all the three definitions agree with the idea that a real system is represented by a

Table 1. Expressions used to define the relation between mathematics and physics.

The place of Mathematics in Physical sciences	Frequency	Percent
Indispensable	10	42
Necessary	7	29
Useful	6	25
No response	1	4
The role of Mathematics in the Physical sciences		
Tool	13	54
Language	4	17
Model	3	12
No response	4	17
A model is		
A simple representation of physical phenomena	7	29
A *maquette* (scale model) representing the real system	7	29
A material allowing to simplify the physical phenomena	5	21
No response	5	21

model, only the first one is acceptable. Actually, these definitions imply that mathematics exhibits no modeling property in students' minds as it was noted earlier. In other words, it is considered that this situation is closely associated with students' perceiving mathematics as a tool in physics rather than a model.

The student teachers' abilities to model physical phenomena by using differential equations

The analysis of the answers for the second part of the data collecting tool revealed that students developed different techniques to obtain a mathematical model:

Correct techniques (CT)

Referring to a physical law in order to deduce the mathematical model representing the physical system proposed. In this technique students, in line with the theoretical approach to construct a model, first determine a physics law which is valid for the given system and construct a mathematical model representing this system based upon the properties of the given system.

Makeshift technique (MT)

Focusing on some familiar formulas of physics to deduce the mathematical model, students try to form a model based on certain relations associated with the given system. Considering modeling process, this technique can be defined as a student's own way to obtain a relation more than a model constructing technique. Indeed, it consists of student's alteration between certain relations associated to the given system that he/she previously memorized. Generally, this technique employs the knowledge obtained in the secondary school.

Heart technique (HT)

Describing the necessary formula without modeling the proposed system. With this technique, a student contents with writing a previously memorized relation that is; he/she does not form a model. This technique is not based necessarily on the modeling of the physical system but on good memory. The answers provided according to this technique contain neither justification nor explanation. Examining the techniques used by the students, they are all associated with a mainstream physics relation or law. These techniques rely on theoretical approach and are related to a physics law or relation, however, only "Correct Technique" has the correct attributions. In Table 2 the techniques used by students to obtain a mathematical model were given with the related frequencies.

Table 2 tells that majority of the students did not answer the second question related to the experimental approach, unlike the rest of the questions related to theoretical approach, which were answered by the majority of the students. Additionally, Table 2 shows that the second technique requiring some physical formulae is the most often used technique by the student teachers and the third technique required remembering the necessary formulae that come in the second place. This Table also shows the limited utilization of the first technique, which is the only technique to achieve the correct answers. The techniques used by the student teachers

Table 2. The techniques used by students to obtain a mathematical model.

	CT	MT	HT	No response
Problem 1	2	14	8	-
Problem 2	-	2	2	20
Problem 3	4	4	8	8
Total (%)	6 (8)	20 (28)	18(25)	28(39)

are presented below and illustrated with some answers:

Correct technique (CT): The analysis of the responses favoring this technique shows that the student teachers manage to correctly identify the physical law required to arrive at the correct answer. A representative response is cited below:

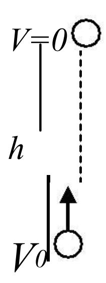
$$U_R + U_L = 0$$

$$i.R + i.r + L.\frac{di}{dt} = 0$$

(The answer given to Problem 3 by Student Teacher 2).

From this answer, one can identify an absolute success concerning the choice of physical law necessary to establish the mathematical model and the realization of the necessary steps to achieve it. However none of the students finding the differential equation representing the given system was able to complete the answer by finding the solution functions of the differential equation.

Makeshift technique (MT): Analysis of the responses using this technique shows that the student teachers did not attempt to model the system proposed but they wrote a known formula and deduced the formula required. The answer below is a representative example of this kind of answer:



$$2.g.h \quad V^2 = V_0^2 \quad 2.g.h$$

(The answer given to Problem 1 by Student Teacher 10).

For the first question, the student teachers made use of the fundamental law of dynamics ($F = ma$) to deduce the differential equation of the system. But the answer cited above shows that the student teachers did not try to find the differential equation but they obtained the expression of the velocity of the object.

By heart technique (HT): As mentioned previously, according to this technique, the student predicts the formula which would be found after the modeling process. What is given below is an example of this type of answers:

$$h = V_0.t + \frac{1}{2}.g.t^2$$

$$V^2 = V_0^2 \quad 2.g.h$$

(The answer given to Problem 1 by Student Teacher 12)

CONCLUSION AND DISCUSSION

The relationship between mathematics and physical science goes back to over four centuries. With this in mind, this study investigated the student teachers' perception of the characteristics of the relationship between mathematics and physics and their abilities to model physical phenomena by using differential equations.

The results of this study reveal that candidate physics teachers state the role of mathematics in physics in three different ways as: indispensable, necessary, and useful. This shows that they are informed about the strong relationship between mathematics and physical sciences.

Foundation and development of theories in physics is based on associating the reality and mathematical theories (Rumelhard, 1997). For this reason a formalization process is needed for modeling and therefore, mathematics plays an important role for physicists. The findings of the present study prove that two basic ideas, language and tool, are valid for candidate physics teachers. This result is line with the result of Rumelhard (1997) who revealed the same roles of mathematics in biology, another science branch related to mathematics. However according to Lange (2000), if mathematics helps to analyze, define and represent a phenomenon, it also let's anticipating, deciding, and making explanations about the same phenomenon. For this reason there have been a complex relation between mathematics and the other sciences and characterizing this relation as a tool is quite a reductive categorization (Lange, 2000). Backed with the data, it can be said that the candidate physics teachers behaved pretty reductive in characterizing the relationship between physics and mathematics and they exploit mathematics as a simple tool to use in problems they face. This result was also supported by the data of the present study about the modeling skills of the candidate teachers (that is, data obtained from the second part of the data collection tool). Because, the available data showed that the candidate teachers had difficulties in obtaining a mathematical model representing a given system. This result supports the result of Buty (2000) saying that, the principal problem of students in modeling process is related to associating the relation between the situation to be modeled and the model.

According to many researchers including Chevallard (1989); Henry (2001); Borremeo (2006), relating the situation to be modeled and the model (that is, forming a model) is one of the fundamental steps of the modeling process. However, the results of this study revealed that the prospective physics teachers had serious difficulties in forming a model. In addition, this study showed that, while forming a model, the students favored both “Makeshift Technique” which consisted of focusing on some familiar formulas and “By Heart Technique” which consisted of remembering the necessary formula without modeling the proposed system.

Another result of this study could be announced by considering textbooks analysis. While the theoretical approach, assumed as related to the model forming step, were rarely included, no traces of the experimental approach was founded in the curriculum. This was thought to have some adverse effects on student achievement because the findings of this study stated that while majority of the students attempted to reply the questions related to theoretical approach, they left unanswered the question about the experimental approach.

Although, students fail in applying the experimental approach, one can notice that it does not imply extremely sophisticated knowledge and it requires working on the experimental curve. In other words, it is based on laboratory works which have an important part in physics. To conclude, physics has to utilize mathematics, the fruitful language of the nature, to explain the natural phenomena. Considering the necessity of two essential components (that is, mathematics and laboratory works) for physics, it can be said that the experimental approach has an essential role to form the effective links between these components. This clearly states the necessity of including the modeling, which takes a tiny emphasis in the curriculum, and the experimental approach of forming a model, completely ignored in the present curricula.

As a recommendation of this study, it can be said that model and modeling process should be tackled as an object and model teaching should not content with model as a tool.

REFERENCES

Arfken G (1985). Mathematical Methods for Physicists. Second edition, Academic press, New York.

Berry JS, Houston SK (1995). Mathematical Modelling. London: Edward Arnold.

Blum W (2002). ICMI Study 14 : Applications and modelling in Mathematics Education-Discussion Document. Educ. Stud. Math. 51(1-2): 149-171.

Borremeo FR (2006). Theoretical and empirical differentiations of phases in the modelling process. Zentralblatt für Didaktik der Mathematik 38(2): 86-95.

Buty C (2000). Etude d’un apprentissage dans une séquence d’enseignement en optique géométrique à l’aide d’une modélisation informatique. Ph.D. Thesis, Université Lumière Lyon 2, Lyon, France.

Chaachoua A, Saglam A (2006). Modelling by differential equations. Teach. Math. Appl. 25(1): 15-22.

Chevallard Y (1989). Le passage de l’arithmétique a l’algèbre dans l’enseignement des mathématiques au collège. Perspectives circulaires : La notion de modélisation, Petit X 19: 43-72.

Chevallard Y (1998). Analyse des pratiques enseignants et didactique des mathématiques : L’approche anthropologique, Actes l’U.E. de la Rochelle.

Cullin M, Crawford BA (2002). Using Technology to Support Prospective Science Teachers in Learning and Teaching About Scientific Models. Contemporary Issues in Technology and Teacher Education [Online serial] *2*(4): 408-425.

Douady R (1986). Jeux de cadre et dialectique outil-objet. Recherche en didactique des mathématiques 7(2): 5-31.

Edwards CH, Penny DE (2005). Differential equations with modeling and computing, Translated by Ömer Akın, Ankara: Palme Yayinlari.

Garcia FJ, Gascon J, Ruiz-Higueras L, Bosch M (2006). Mathematical modelling as a tool for the connection of school mathematics. Zentrablatt für didaktik der Mathematic 38(3): 226-246.

Gobert JD, Buckley BC (2000). Introduction to Model-Based Teaching and Learning in Science Education, Int. J. Sci. Edu. 22(9): 891-894.

Guillon A (1995). Démarches scientifiques en travaux pratiques de physique de DEUG à l’université de Cergy-Pontoise, Didaskalia 6: 113-127.

Helfgott M (2004). Two Examples from the Natural Sciences and their Relationship to the History and Pedagogy of Mathematics. Mediterranean J. Res. Math. Educ. 3(1-2): 147-166.

Henry M (2001). Notion de modèle et modélisation dans l’enseignement in Henry M. (Ed.) *Autour de modélisation en probabilité* (149-159), Besançon : Commission inter-IREM Statistique et Probabilité.

Karao lu B (1994). Fizik ve Mühendislikte Matematik Yöntemleri (Mathematical methods in Physics and Engineering), Bilgitek yayinlari: Istanbul.

Lange JM (2000). Mathématisation de la biologie, discipline de la recherche et discipline enseignée : état de lieu et enjeux didactiques, Ph.D. Thesis, ENS Cachan.

Marton F, Yang Pong W (2005). On the Unit of Description in phenomenography, Higher Educ. Res. Dev. 24(4): 335-348.

Mevarech ZR, Kramarski B (2004). Mathematical Modeling and Meta-cognitive Instruction, ICME 10: 10^{th} International Congress on Mathematical Education, www.icmeorganisers.dk/tsg18/S32MevarechKramarski.pdf, Reviewed at 20/04/2009.

Rodriguez R (2007). Les équations différentielles comme outil de modélisation mathématique en classe de Physique et de Mathématiques au lycée : une étude de manuels et de processus de modélisation d’élèves en Terminale *S*. Ph.D. Thesis, Université Joseph Fourier, Grenoble-France.

Rumelhard G (1997). Travailler les obstacles pour assimiler les connaissances scientifiques. Aster 24: 13-35.

Saglam A (2004). Les équations différentielles en mathématiques et en physique: Etude des conditions de leur enseignement et caractérisation des rapports personnels des étudiants de première année d’université à cet objet de savoir. Ph.D. Thesis, Université Joseph Fourier, Grenoble-France.

Tzanakis C (2002). On the Relation Between Mathematics and Physics in Undergraduate Teaching, 2^{nd} International Conference on the Teaching of Mathematics July 1-6, Greece.