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*Full Length Research Paper*

# Portfolio value at risk with Copula-ARMAX-GJR-GARCH model: Evidence from the gold and silver futures

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**In the article, we construct the copula-based VaR-ARMAX-GJR-GARCH model. The purpose is to examine the strategic commodities comovements and directional relationships with these variables, as well as estimating the VaR of a gold and silver portfolio. Based on our empirical results, we conclude that the crude oil for the gold and silver price in Comex and Tocom market is both a significant and positive sign whether before or during uptrend. As to US/Japan yen exchange rate, there is still no consistent result. That is to say there is no evidence that an influence of the variable to gold and silver futures exists. In addition, the time-varying SJC copula, which allows for different dependence in the tails, produced the best result regardless of being before or during uptrend. Furthermore, concerning risk management, copula-based models more accurately assess portfolio risk.**

**Key words:** Copula function, value at risk, Kendall's tau, Joe-Clayton copula.

## INTRODUCTION

Commodity derivatives markets have experienced great growth in recent years especially in Gold and Silver futures. The greatest trading volumes of gold and silver futures is COMEX in U.S. and TOCOM in Japan. Thus, they are relatively representative of the world's gold and silver markets. Specifically, investors can apply the related results to hedge or arbitrage in the gold and silver spot markets. Nevertheless, it is useful for investors, speculators, and decision makers to understand gold and silver futures market risk.

Traditionally, gold has played a considerable role during times of political and economic crises and equity market crashes. Faff and Chan (1998) report that gold stocks play an important role as a hedge against other stocks. The authors report that investment in gold provides an effective hedge against inflation and political instability. Taylor (1998) states that both gold and silver along with platinum have provided a short and long run hedge against inflation. In some ways gold in particular can be seen as the ultimate risk-less asset, despite its volatility. To know the exact extent of risk for gold and silver can have a substantial impact on valuation, investment decisions, and risk management, thus we calculate the value-at-risk in this paper. To evaluate portfolio risk, first calculating the correlation between price or volatility behaviours that occur between a portfolio's Gold or Silver is crucial for properly estimating the value-at-risk amount. However, restrictions on the joint distributions of Gold or Silver within the portfolio might decrease the performance of the value-at-risk estimation.

The Hammoudeh et al. (2009) study indicates that commodity prices tend to rise and fall together. There is also a recent belief that oil price is the most volatile, and may lead to price procession. It is not surprising that commodity prices may move in union because they are typically influenced by common macroeconomic factors such as crude oil and the U.S. exchange rate. For example, increased commodity prices usually fuel higher inflation expectations, giving rise to monetary policy tightening and resulting in increases in interest rates. Raised interest rates, increase the price of precious metals, gold in particular, which could lead to money flight to gold safety and away from the dollar, resulting in dollar depreciation.

It would be interesting and useful to decide whether the strategic commodities (oil, gold, silver), and the U.S. exchange rate move together over time, and also to understand the moving forces behind each one of them. It would also be worthwhile to see how the precious metals, gold and silver, move over time in the presence of oil

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and exchange rates. Are these assets substitutes for the same type of risk or are they different in the sense that they can be used in portfolios that diversify away risk? We also need to find out how the U.S. dollar behaves in the presence of precious metals prices. Would they also fly to the safety of gold when the dollar depreciates? Oil price movements and exchange rates do not always offset each other.

The exact risk depends on the structure of the foreign trade, basically the source of their imports. Oil price and the dollar exchange rate represent overall mutually independent risk for agents outside the United States, but with varying intensity and direction. The historical pattern is that in some cases, movements of the dollar exchange rate enhance oil price movements, but that in other cases, the dollar exchange rate tends to lessen oil price changes. The lack of a clear pattern shows that oil prices and currency exchange rates have overall different driving forces-sometimes they agree; sometimes they differ.

One of the early studies on price comovements is Pindyck and Rotemberg (1990). This study finds excess comovements in prices of seven unrelated commodities. Other research, however, finds less “excessive” comovement in prices of commodities (Deb et al. 1996; Palaskas and Varangis, 1991; Trivedi, 1995). Using the concordance measures, Cashin et al. (1999) analyze the veracity of the comovements concerning the prices of 17 related an unrelated commodities. In particular, they find no evidence of such comovement in the prices of related commodities. On the other hand, Ciner (2001) finds that the stable long-run relationship between gold and silver on the Tokyo Commodity Exchange disappeared in the 1990s, and more recently, they have their own separate markets as they are considered to have different economic uses. However, Tully and Lucey (2007) suggest that while there are periods when the relationship between those two precious metals is weak, overall, a stable relationship prevails.

Our study is more focused and timelier than previous studies due to more concentrating on strategy. These studies, however, use daily data and, for the most part, do not include all of our daily commodity-relevant macro financial variables, particularly the exchange rate. The role of the U.S. dollar exchange rate has become very prominent in affecting and being affected by the price of oil and the prices of precious metals. The Organization of the Petroleum Exporting Countries (OPEC) increased oil prices to compensate for the falling purchasing power of their dollar-denominated oil revenues. As a result, investors move to the safety of gold and away from the falling dollar. All these considerations have been helped by the current availability of adequate and lengthy daily time series for both commodity and macro-financial variables, which figures highly in this study.

Therefore, in this article, we primarily examine the dynamic movements of these strategic commodities, which as indicated above are among the most traded in the world commodity markets. Second, we follow the literature and explore their individual directional relationships with one commodity-relevant macro financial variable: exchange rates that have not received much attention in the academic literature on commodities. However, it is difficult to discuss the dynamic correlation between assets of portfolio. In the article, we will consider the copula function to investigate the correlation. copula function was widely used in financial econometrics and risk management. For example, Palaro and Hotta (2006) used conditional copula to estimate VaR. Junker et al. (2006) discussed the nonlinear term structure dependence and risk implication based on copula function. Hu (2006) proposed a mixed copula modele model that it can capture various patterns of dependence structures. Rodriguez (2007) modeled dependence with switching-parameter copulas to study financial contagion. Chiou and Tsay (2008) addressed a copula-based approach to option pricing and risk assessment. Hsu et al. (2008) proposed copula-based GARCH models for the estimation of the futures optimal hedge ratio. Manner et al. (2009) used copula models with time-varying dependence structure.

The new look comes as a result of (a) examining the comovements and directional relationships of strategic commodities with oil, given the opportunity to lead the procession; (b) augmenting the system to include the U.S. dollar exchange rate against the Japanese yen to reflect the recent realities in the world commodity and financial markets; (c) using daily data instead of monthly or quarterly data to enhance the information content and processing mechanism; and (d) using value at risk (hence VaR) based on the copula-ARMAX-GARCH Model approach. Empirically, this paper uses gold and silver future data from COMEX and TOCOM, as well as WTI crude oil and US/Japan Yen spot rates (hence US/JYI). This study’s sample period covers twenty years, from January 1990 to December 2009. We conclude that the crude oil for the gold and silver price in Comex and Tocom market is both a significant and positive sign whether before or during uptrend. The Japanese yen is not significant in most cases. The VaR is based on the fact that a time-varying symmetric Joe-Clayton copula and GJR-GARCH models can outperform other VaR models. This shows that time-varying copula-based methods can gain insights into value-at-risk (VaR) and other risk measurements.

### COPULA-ARMAX-GJR-GARCH based VaR Model

The portfolio’s VaR at time *t*, with significance level α, where $\alpha \in (0,1)$, can be defined as:

$$P\{X_{P,t} \leq VaR_t(\alpha)\} = \alpha \tag{1}$$

This means that we are 100(1-α)% confident that the loss in the given period will not be larger than $VaR_t(\alpha)$.

Because of the observed negative skewness, we decided to filter the returns with the semi-parametric method. In specifying the bivariate model we must specify the two models for the marginal variables and the model for the conditional copula. The models for the univariate variables must take into account the variables characteristics. Return series have been successfully modeled by the ARMAX(1,0,0)-GJR-GARCH(1,1) model assuming Gaussian residuals. Assume two return series $r_{1,t}$, $r_{2,t}$,..., follow a ARMAX(1,0,0)-GJR-GARCH(1,1)

$$r_{i,t} = c + s r_{i,t-1} + \varphi X_{i,t} + \varepsilon_{i,t}, \quad i=1,2 ; t=1,2,...,T \qquad (2a)$$

$$\varepsilon_{i,t} = \sqrt{h_{i,t}}\, z_{i,t} \qquad z_{i,t} \sim N(0,1) \qquad (2b)$$

$$h_{i,t} = \sigma^2_{i,t} = \omega + \alpha\varepsilon^2_{i,t-1} + \beta h_{i,t-1} + \gamma\varepsilon^2_{i,t-1} d_{i,t-1} \qquad (2c)$$

Where,

$$d_{i,t} = \begin{cases} 1, if & \varepsilon_{i,t} < 0 \\ 0, if & \varepsilon_{i,t} \geq 0 \end{cases} \qquad (2d)$$

$$(z_{1t}, z_{2t}) \sim C_t(F(z_{1t}), F(z_{2t})) \qquad (2e)$$

Where, $X_{i,t}$ is an explanatory regression matrix. $z_{i,t} = \varepsilon_{i,t} / \sqrt{h_{i,t}}$ is the conditional distribution of standardized innovations. In this study, we set i=1, 2. The distribution of the innovation vector $z_t = (z_{1t}, z_{2t})$ is modeled by copula. $C_t$(……,.). Here, C was modeled by Normal, student-t, Clayton-Copula, Gumbel Copula and Frank Copula function and time varying copula (time varying normal copula and Joe-Clayton copula) respectively.

We further calculate the variance-covariance of portfolio ComexGold-TocomGold, ComexSilver-ComexSilver $\sigma^2_{1,2,t}$ due to bivariate GARCH models(Engle,2002). The model is given by:

$$\sigma^2_{1,2,t} = \begin{bmatrix} \sigma^2_{1,t} & \sigma_{1,2,t} \\ \sigma_{1,2,t} & \sigma^2_{2,t} \end{bmatrix} = \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix} \begin{bmatrix} 1 & \rho_{1,2} \\ \rho_{1,2} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix}$$

Consider the weight $w_1$ and $w_2$ of portfolio asset 1 and asset 2, the variance-covariance will be:

$$\sigma^2_{1,2,t} = w_1^2\sigma^2_{1,t} + w_2^2\sigma^2_{2,t} + 2w_1w_2\rho_{1,2,t}\sigma_{1,t}\sigma_{2,t} \qquad (3)$$

Where, $w_1$=$w_2$=0.5, we set equal weight in ComexGold(ComexSilver) and TocomGold(TocomSilver)

$\sigma^2_{1,t}$ : condition variance of ComexGold (ComexSilver)

$\sigma^2_{2,t}$ : condition variance of TocomGold (TocomSilver)

$\rho_{1,2}$ : the constant correlation coefficient between 1,2 , here we set

$\rho_{1,2,normal-copula}$ = Normal-copula Kendall's tau; $\rho_{1,2,t-copula}$ = t-copula Kendall tau; $\rho_{1,2,Clayton-copula}$ =Clayton-copula Kendall tau ; $\rho_{1,2,Gumbel-copula}$ =Gumbel-copula Kendall tau;

$\rho_{1,2,Frank-copula}$ =Frank-copula Kendall tau;

$\rho_{1,2,Normal-DC-copula}$ =Normal-DC-copula Kendall's tau;

$\rho_{1,2,SJC-L-copula}$ =SJC-L-copula Kendall's tau; $\rho_{1,2,SJC-U-copula}$ =SJC-L-copula Kendall tau.

The time-varying normal copula tau function is given:

$$\rho_{1,2,t} = \tilde{L}[\omega_\rho + \beta_\rho\rho_{1,2,t-1} + \alpha_\rho \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(u_{t-j}).\Phi^{-1}(v_{t-j})] \qquad (4)$$

Where $\rho$ is normal Kendall' tau, $\tilde{L}(x) = \frac{1-e^{-x}}{1+e^{-x}}$, the modified logistic function ; $\Phi^{-1}$ is the inverse of the standard normal CDF.

The symmetric Joe-Clayton copula (Patton ,2006) is given by:

$$C_{JC}(u, v|\tau_U, \tau_L) = 1-(\{[1-(1-u)^k]^{-\gamma} + [1-(1-v)^k]^{-\gamma} - 1\}^{-(1/\gamma)})^{(1/k)} \qquad (5)$$

Where

$$k = 1/\log_2(2-\tau_U), \qquad \gamma = -1/\log_2(\tau_L),$$
$$\tau_U \in (0,1),, \qquad \tau_L \in (0,$$

$\tau_U$ and $\tau_L$ are the coefficients of upper and low tail dependence, respectively.

**Upper tail dependence**

$$\tau^u_t = L[\omega_u + \beta_u\tau^u_{t-1} + \alpha_u \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|]$$

**Lower tail dependence**

$$\tau^L_t = L[\omega_L + \beta_L\tau^L_{t-1} + \alpha_L \frac{1}{10}\sum_{j=1}^{10}|u_{t-j} - v_{t-j}|]$$

$L(x) = \frac{1}{1+e^{-x}}$ is the logistic function.

Then, we have the eight variance-covariance VaR models, they are

$$VaR_{normal-copula} = Z_\alpha \cdot \sigma_{1,2,t,normal-copula} \cdot \sqrt{T} \qquad (6a)$$

$$VaR_{t-copula} = Z_\alpha \cdot \sigma_{1,2,t,t-copula} \cdot \sqrt{T} \qquad (6b)$$

$$VaR_{Clayton-copula} = Z_\alpha \cdot \sigma_{1,2,t,Clayton-copula} \cdot \sqrt{T} \qquad (6c)$$

$$VaR_{Gumbel-copula} = Z_{\alpha} \cdot \sigma_{1,2,t,Gumbel-copula} \cdot \sqrt{T} \quad (6d)$$

$$VaR_{Frank-copula} = Z_{\alpha} \cdot \sigma_{1,2,t,Frank-copula} \cdot \sqrt{T} \quad (6e)$$

$$VaR_{Normal-DC-copula} = Z_{\alpha} \cdot \sigma_{1,2,t,Normal-DC-copula} \cdot \sqrt{T} \quad (6f)$$

$$VaR_{SJC-L-copula} = Z_{\alpha} \cdot \sigma_{1,2,t,SJC-L-copula} \cdot \sqrt{T} \quad (6g)$$

$$VaR_{SJC-U-copula} = Z_{\alpha} \cdot \sigma_{1,2,t,SJC-U-copula} \cdot \sqrt{T} \quad (6h)$$

Where α= 5, 2.5 and 1% significance level, T=1(one day).

In order to evaluate the performance of this copula-based portfolio VaR model, we apply the proportion failure test method Kupiec's (1995) . Kupiec's tests are based on likelihood ratios, and on the assumption that VaR should exhibit a conditional or unconditional coverage equal to the given significance level, 5, 2.5 and 1%, respectively.

Kupiec (1995) proposed the likelihood ratio test based on the fact that the number of exceeding VaR among the samples of size *T* follows a binomial distribution, with the probability proportional to $(1-p)^{T-N}p^{N}$ and hence the likelihood ratio (LR) statistic follows a chi-square distribution with degree of freedom 1.

$$LR_{uc} = -2\ln(1-p)^{T-N}p^{N} - 2\ln(1-N/T)^{T-N}(N/T)^{N} \sim \chi^2(1) \quad (7)$$

$$H_o : p = \hat{p};\ H_1 : p \neq \hat{p}$$

Where p is the desired significance level, N is the number of times in the sample when the VaR forecast is exceeded, and T is the sample number.

## RESULTS AND ANALYSES

The analysis of this study is to investigate the information transmission between gold and silver future in COMEX and TOCOM. The data set consists of futures contract on gold, silver, as well as the spot rate of WTI crude oil and US/JYI exchange rates. The sample period for the study covers twenty years, from January 1990 to December 2009. All of the analysis is conducted on return data. Table 1 reports the return's descriptive statistics. It shows the high Kurtosis and negative skewness pattern. From the price time series of Figure 1, we recognize that the price of gold and silver futures for the two markets began a sustained uptrend on 2 April 2001. That is, there may be structural breaks in the gold and silver futures markets. We used a Chow test (1960) to verify this hypothesis and the results show structural breaks indeed exist. Therefore, we divided the sample period into two subsets (pre- and post-April 2, 2001) to examine if price behavior exhibits different characteristics during the point where an uptrend in gold and silver futures began. The period before the date is labeled as "before the uptrend." The data period includes January 1990 to March 2001.

On the other hand, after the date is described as "during the uptrend." In the data period containing April 1990 to December 2009 there are 2647 and 2077 gold and silver futures price observations before and during the uptrend, respectively.

Figure 2 plots the trend of crude oil and the Japanese yen (on log scare). Comparing Figures 1 and 2, we find that the crude oil and the Gold and Silver price has the same trend expect for September 1998 to May 2001. In that time, the gold price went down while crude oil went up. Yet, the Japanese yen went on its way. Figure 3 shows the gold and silver futures return series during the entire period. Table 1 reports summary statistics for these series. We observe that none of the series seem to follow either the Gaussian or Student-t distribution. The mean returns are positive during the entire period. High kurtosis and negative skewness (skewed left and / or fat tail) characteristics are also visible in these series.

As mentioned earlier, we consider the macro financial variables crude oil and Japanese yen in the ARMAX conditional mean equation. Table 2 presents the results before uptrend. As can be seen, crude oil for the gold and silver price in Comex is significant and positive, which indicates that crude oil increases. Therefore the hedge needed will lead to a gold and silver price increase. As for the Japanese yen, there is still no consistent result. It is significant on the TocomGold and ComexSilver at a 5% significance level. The ComexGold and ComexSilver variable have negative and positive signs on TocomGold and TocomSilver, respectively. In the condition variance equation, we employ the GJR-GARCH(1,1) model. The parameters are all significant at both 1 and 5% significance levels. This also reveals that the GJR-GARCH(1,1) fits the asymmetric variance very well when caused by good or bad news.

For comparison purposes, we utilize five constant Kendall's tau copula functions. They are: Normal Copula, Student Copula, Clayton Copula, Gumbel Copula and Frank Copula, respectively. Table 3 reports the estimated results, including Kendall's tau and other criteria, including Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood value (LL). By means of the above criteria, in Panel A: ComexGold-TocomGold, we will choose the Kendall's tau of Frank copula named as the $\rho_{i,j,Frank-copula}$ in Equation (3). As for Panel B: ComexSilver-TocomSilver, the optimal Kendall's tau is named $\rho_{i,j,Gumbel-copula}$ in Equation (3). In addition, we further use two time-varying Copula func-tions in Equation (3). They are a normal dynamic Copula function known hereafter as Normal DC Copula and Symmetrized Joe-Clayton Copula (SJC), (Patton, 2003). The SJC copula has two parameters, $\tau_U$ and $\tau_L$, which are the coefficients of upper and low tail dependence, respectively. With these the Kendall's tau portfolio variance-covariance will be calculated according to Equation (3). We can further obtain the portfolio's VaR

**Table 1.** Descriptive statistics of the four future returns (full period).

| Item | Mean | Std. | Kurtosis | Skewness | JB-stat | P-value |
|---|---|---|---|---|---|---|
| ComexGold | 0.0002 | 0.0105 | 11.0211 | -0.0356 | 12662.287 | 0.0010* |
| ComexSilver | 0.0002 | 0.0182 | 10.5563 | -0.4777 | 11416.119 | 0.0010* |
| TocomMGold | 0.0001 | 0.0115 | 7.0358 | -0.3431 | 3297.900 | 0.0010* |
| TocomSilver | 0.0001 | 0.0173 | 10.4395 | -0.0258 | 10892.169 | 0.0010* |

P-value is the probability that the data comes from the Normal distribution, according to Jarque-Berra normality test

**Table 2.** Results from the ARMAX (1)–GJR-GARCH(1,1) model (before uptrend).

| | | | | |
|---|---|---|---|---|
| | **Conditional mean equation ARMAX(1,0,0)** | | | |
| | **ComexGold** | **TocomGold** | **ComexSilver** | **TocomSilver** |
| c | -0.00011993 | -8.4142e-005 | -0.00017498 | -0.00021771 |
| | (-0.8948) | (-0.4764) | (-0.6707) | (-0.8328) |
| Crude oil | 7.3023e-005** | 0.0004525* | 8.6853e-006** | 0.00050113* |
| | (2.3200) | (1.3844) | (2.0218) | (1.3377) |
| US/JYI | -7.2886e-005 | 0.00027525** | -0.00044745** | 0.00018062 |
| | (-0.5308) | (1.6709) | (-1.6348) | (0.7368) |
| | **Conditional variance equation GJR-GARCH(1,1)** | | | |
| ω | 4.3004e-007*** | 2.9709e-006*** | 1.3079e-006*** | 2.3412e-006*** |
| | (12.4903) | (7.1742) | (4.5133) | (5.2793) |
| α | 0.95145*** | 0.88571*** | 0.96485 *** | 0.93728*** |
| | (484.3744) | (96.2365) | (313.2067) | (189.0608) |
| β | 0.069673*** | 0.099232*** | 0.047371*** | 0.052079*** |
| | (21.2222) | (9.2872) | (12.3244) | (8.9792) |
| γ | -0.048364*** | -0.023867** | -0.034239*** | 0.0032253** |
| | -(13.0360) | (-1.9956) | (-7.9319) | (1.4680) |
| LL | -14.9687 | -87.5017 | -98.4402 | -162.1293 |

The estimated parameters correspond to equations 2(a) to 2(d). LL corresponds to the log - likelihood function value; The t values are in parenthesis; The * **, ** stand for 10, 5, 1% , respectively; Model states as (6a), (6b),(6c),(6d),(6e),(6f),(6g),(6h).

**Table 3.** The Kendall's tau and fitness of copula functions (before uptrend).

| | | | | | |
|---|---|---|---|---|---|
| | **Panel A: ComexGold-TocomGold** | | | | |
| | **Normal Copula** | **Student Copula** | **Clayton Copula** | **Gumbel Copula** | **Frank Copula** |
| Kendall tau | 0.0687 | 0.0489 | 0.0631 | 0.0793 | 0.0532 |
| LL | -15.4529 | -87.6704 | -21.3788 | -38.4173 | -7.7394 |
| AIC | -30.9057 | -175.3407 | -42.7576 | -76.8339 | -15.4784 |
| BIC | -30.9054 | 175.3406 | -42.7573 | -76.8314 | -15.4773 |
| | **Panel B: ComexSilver-TocomSilver** | | | | |
| Kendall tau | 0.1886 | 0.1885 | 0.1914 | 0.1484 | 0.1929 |
| LL | -117.8769 | -178.3053 | -146.9199 | -92.8115 | -104.4093 |
| AIC | -235.7535 | -356.6103 | -293.8389 | -185.6226 | -208.8173 |
| BIC | -235.7529 | -356.6097 | -293.8362 | -185.6219 | -208.8134 |

LL is the copula estimation log-likelihood value.,(AIC) (Akaike, 1974), which is defined as $AIC(M)$ = 2 LL + 2T;where T is the number of parameters being estimated and hat denotes the maximum likelihood estimates. Bayesian information criterion (BIC),( Schwarz,1978).

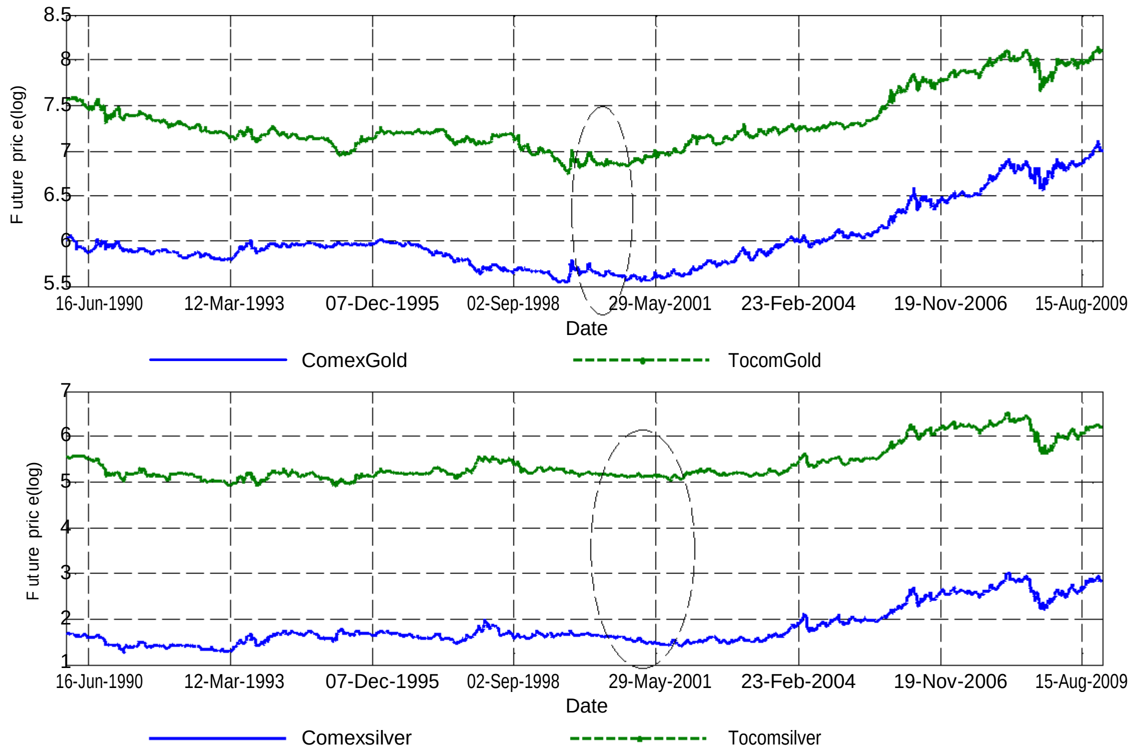


**Figure 1.** The price trend of COMEX and TOCOM gold futures (The COMEX gold futures and silver futures are quoted in US dollars; while the TOCOM gold futures and silver futures are quoted in Japanese yen).

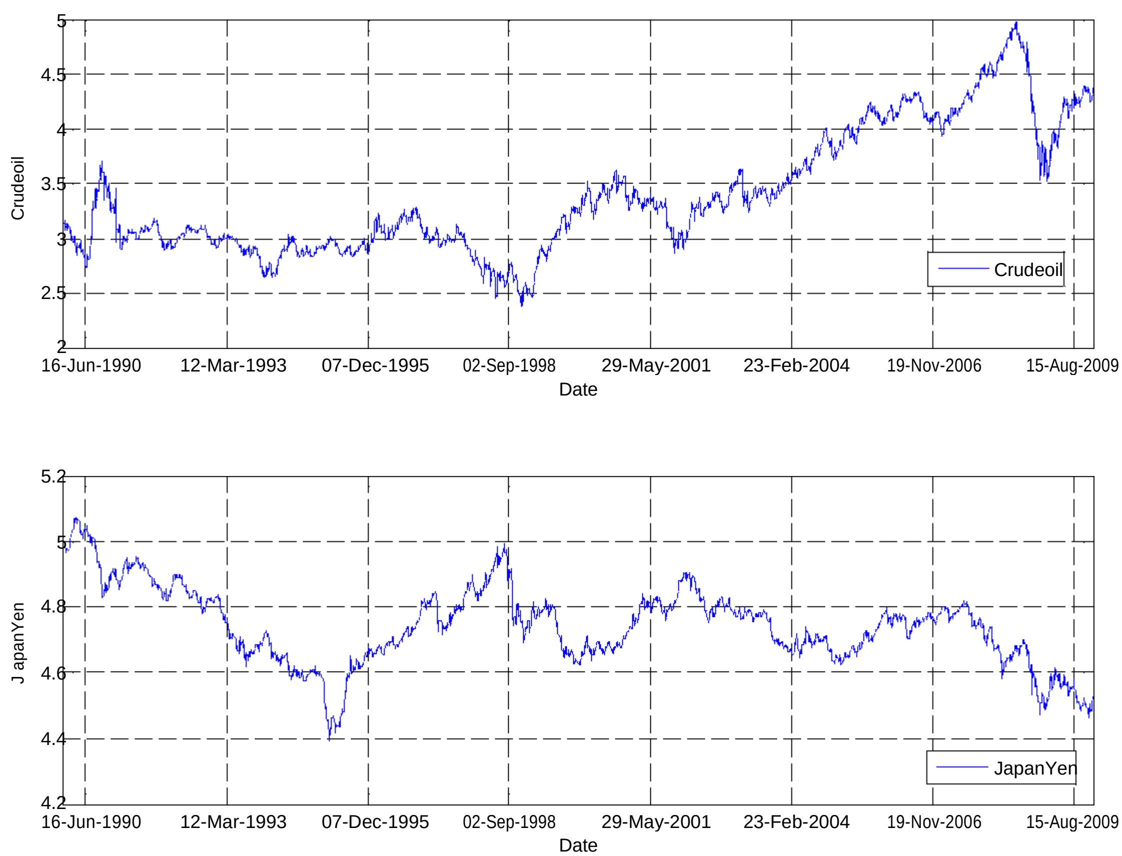


**Figure 2.** The crude oil and Japanese Yen price trend (on log scale).

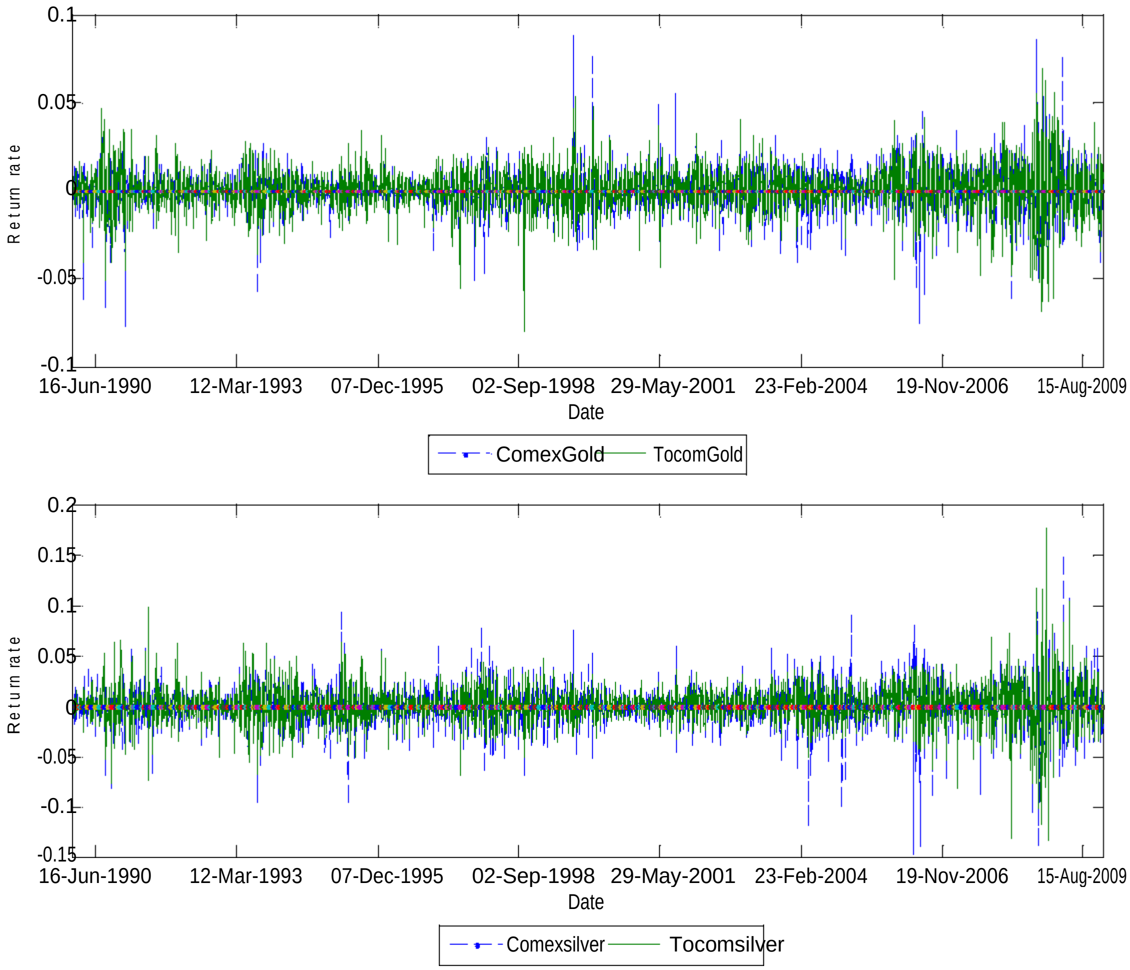


**Figure 3.** The returns trend of COMEX and TOCOM gold and silver futures.

for ComexGold-TocomGold,ComexSilver-TocomSilver through Equations (6a) to (6h).

Table 4 summarizes the VaR results under the 5, 2.5 and 1% significance levels, two back testing criteria (Failure rate and LRuc) and four copula functions. By contrast in Table 4, the closer the significance level and failure rate, the more accurate the VaR models.

Nevertheless, the LRuc is not significant at different significance levels; the more reliable are the VaR models. According to the two criteria, in Panel A: ComexGold-TocomGold, $VaR_{SJC\_tauU}$ can outperform other three copula functions. Its failure rate is 0.0465, 0.0295 and 0.0170 at the 5, 2.5 and 1% significance levels, respectively. Nonetheless, $VaR_{SJC\_tauL}$ is the best model among the four VaR models at Panel B: ComexSilver-TocomSilver portfolio.

Figure 4 shows the time varying Joe-Clayton tau. The left panel is the ComexGold-TocomGold portfolio, and right panel is ComexSilver-TocomSilver portfolio. It is clear that the correlation between the two assets is time varying, especially in the ComexSilver-TocomSilver portfolio. It indicates a larger change during the period. Figure 5 illustrates the shape of portfolio VaR based on SJC-copula at 99, 97.5 and 95% confidence levels. The left panel is the ComexGold-TocomGold portfolio, and the right is the ComexSilver-TocomSilver.

By contrast, Table 5 reports the results during the uptrend. As you can see, the crude oil for the gold and silver price in Comex and Tocom markets are all significant and positive. This indicates the positive correlation between oil price and gold and silver futures price. As for the Japanese yen, there is still no consistent result. It is significant at TocomGold under 5% significance level. The variable for the ComexGold and TocomGold is positive, while being negative on TocomSilver and TocomSilver. Meanwhile in the condition variance equation we also employ the GJR-GARCH(1,1) model. The parameters are all significant at 1%, 5% or 10% significance levels, respectively. This also reveals that the GJR-GARCH(1,1) fits the conditional variance very well during the uptrend. Table 6 reports the estimated copula functions results with the same criteria

**Table 4.** Portfolio VaR (before uptrend).

| Panel A:ComexGold-TocomGold | | | | | | |
|---|---|---|---|---|---|---|
| | α=0.05 | | α=0.025 | | α=0.01 | |
| | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** |
| $VaR_{Frank\_tau}$ | 0.0450(119) | 2.2017 | 0.0302(80) | 2.7908 | 0.0178(47) | 13.0857* |
| $VaR_{NDCC\_tau}$ | 0.0454(120) | 1.2408 | 0.0302(80) | 2.7908 | 0.0178(47) | 13.0857* |
| $VaR_{SJC\_tauL}$ | 0.0442(117) | 1.9340 | 0.0299(79) | 2.4127 | 0.0197(52) | 19.4332* |
| $VaR_{SJC\_tauU}$ | **0.0465(123)** | **0.7040** | **0.0295(78)** | **2.0608** | **0.0170(45)** | **10.8441*** |

| Panel B:ComexSilver-TocomSilver | | | | | | |
|---|---|---|---|---|---|---|
| | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** |
| $VaR_{Gumbel\_tau}$ | 0.0537(142) | 0.7319 | 0.0283(75) | 1.1649 | 0.0189(50) | 16.7711* |
| $VaR_{NDCC\_tau}$ | 0.0533(141) | 0.5901 | 0.0291(77) | 1.7353 | 0.0197(52) | 19.4332* |
| $VaR_{SJC\_tauL}$ | 0.0533(141) | 0.5901 | 0.0310(82) | 3.6239 | 0.0178(47) | 13.0857* |
| $VaR_{SJC\_tauU}$ | **0.0503(133)** | **0.0039** | **0.0276(73)** | **0.7042** | **0.0159(42)** | **7.8233*** |

Total observation is 2,646 , Failure rate expresses the ratio of portfolio loss exceeded the estimated VaR for α = 0.05, 0.025 and 0.01. * denotes significant at 1% significance level.

**Table 5.** Results from the ARMAX (1)–GJR-GARCH(1,1) model (during the uptrend).

| Conditional mean equation ARMAX(1,0,0) | | | | |
|---|---|---|---|---|
| | **ComexGold** | **TocomGold** | **ComexSilver** | **TocomSilver** |
| c | 0.00075094*** | 0.0006022*** | 0.0005977** | 0.00062457** |
| | (3.1901) | (2.6713) | (1.5813) | (1.8530) |
| Crude oil | 0.00089975*** | 0.00051077*** | 0.00026026** | 0.00056769** |
| | (5.5718) | (3.3749) | (1.9595) | (2.1533) |
| US/JYI | 0.00028947 | 0.00057317** | -0.00057561 | -3.7758e-005 |
| | (0.9544) | (1.9749) | (-1.2474) | (-0.0921) |
| **Conditional variance equation GJR-GARCH(1,1)** | | | | |
| ω | 1.0007e-006*** | 1.7101e-006*** | 2.3802e-006*** | 1.7706e-006*** |
| | (3.0054) | (4.0354) | (4.5193) | (3.2511) |
| α | 0.95684*** | 0.90027*** | 0.94096*** | 0.93533*** |
| | (157.7687) | (81.9163) | (211.4635) | (149.4975) |
| β | 0.055407*** | 0.10683*** | 0.076979*** | 0.069391*** |
| | (5.8755) | (6.8666) | (10.8931) | (6.8766) |
| γ | -0.034918*** | -0.029165** | -0.037462*** | -0.015469* |
| | (-3.4706) | (-1.7978) | (-5.4897) | (-1.3148) |
| LL | -209.5336 | -256.6131 | -589.7214 | -785.8453 |

The estimated parameters correspond to equations 2(a) to 2(d). LL corresponds to the log - likelihood function value; The t values are in the parenthesis; The * **, ** stand for 10, 5 and 1% , respectively; Model states as (6a), (6b),(6c),(6d),(6e),(6f),(6g),(6h).

as in Table 3. Under the critera, in Panel A: ComexGold-TocomGold, we will choose the Kendall's tau of Frank copula and named as $\rho_{i,j,Frank-copula}$ in Equation (3). As to Panel B: ComexSilver-TocomSilver, the Kendall's tau of Gumbel copula will be chosen, and named as $\rho_{i,j,Clayton-copula}$ inEquation (3). In addition, we used

**Table 6.** The Kendall's tau and fitness of copula functions (during the uptrend).

| | Normal Copula | Student Copula | Clayton Copula | Gumbel Copula | Frank Copula |
|---|---|---|---|---|---|
| **Panel A:ComexGold-TocomGold** | | | | | |
| Kendall tau | 0.2915 | 0.2902 | 0.2464 | 0.2819 | 0.2982 |
| LL | -225.6260 | -276.6178 | -207.5370 | -228.4798 | -203.8666 |
| AIC | -451.2515 | -553.2352 | -415.0734 | -456.9583 | -407.7304 |
| BIC | -451.2503 | -553.2340 | -415.0716 | -456.9545 | -407.7226 |
| **Panel B:ComexSilver-TocomSilver** | | | | | |
| Kendall tau | 0.457 | 0.501 | 0.4106 | 0.479 | 0.5116 |
| LL | -589.72 | -785.84 | -555.303 | -652.94 | -651.32 |
| AIC | -1179.44 | -1571.68 | -1110.606 | -1305.89 | -1302.64 |
| BIC | -1179.44 | -1571.61 | -1110.604 | -1305.86 | -1302.62 |

LL is the log-likelihood value of copula estimation.,(AIC) (Akaike, 1973) which is defined as $AIC(M)$ = 2 LL + 2T;where T is the number of parameters being estimated and hat denotes the maximum likelihood estimates. Bayesian information criterion (BIC), (Schwarz,1978).

**Table 7.** VaR of portfolio (during the uptrend).

| | α=0.05 | | α=0.025 | | α=0.01 | |
|---|---|---|---|---|---|---|
| **Panel A:ComexGold-TocomGold** | | | | | | |
| | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** |
| $VaR_{Frank\_tau}$ | 0.0568(118) | 1.9621 | 0.0356(74) | 8.5446* | 0.0178 (47) | 13.0857* |
| $VaR_{NDCC\_tau}$ | 0.0564(117) | 1.7002 | 0.0356(74) | 8.5446* | 0.0170(45) | 10.8441* |
| $VaR_{SJC\_tauL}$ | 0.0544(113) | 0.8353 | 0.0337(69) | 5.2456 | 0.0151(40) | 6.0497 |
| $VaR_{SJC\_tauU}$ | 0.0578(120) | 2.5396 | 0.0381(75) | 9.2900* | 0.0178(47) | 13.0857* |
| **Panel B:ComexSilver-TocomSilver** | | | | | | |
| | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** | **Failure rate** | **LRuc** |
| $VaR_{Gumbel\_tau}$ | 0.0588(122) | 5.8470 | 0.0337(70) | 5.8470* | 0.0155(41) | 6.9118* |
| $VaR_{NDCC\_tau}$ | 0.0573(119) | 2.2419 | 0.0318(66) | 3.6228 | 0.0159(42) | 8.7833* |
| $VaR_{SJC\_tauL}$ | 0.0530(110) | 0.3827 | 0.0299(62) | 1.8996 | 0.0151(40) | 6.0497 |
| $VaR_{SJC\_tauU}$ | 0.0583(121) | 2.8550 | 0.0337(70) | 15.8470* | 0.0155(41) | 6.9118* |

Total observation is 2076 ,Failure rate expresses the ratio of portfolio loss exceeded the estimated VaR for α = 0.05, 0.025 and 0.01 and 0.01. * denotes significant at 1% significance level.

three time-varying copula functions in equation (3). With these Kendall's tau, then the portfolio's variance-covariance will be calculated by equation (3). We can obtain the portfolio's VaR for ComexGold-TocomGold, ComexSilver-TocomSilver through equation (6a) to (6h).

Table 7 summarizes the VaR results under the 5, 2.5 and 1% significance levels, two back testing criteria, Failure rate and LRuc, and four Copula functions. The same as before trend, in Panel A: ComexGold-TocomGold, $VaR_{SJC\_tauL}$ can outperform other three copula function. Whereas $VaR_{SJC\_tauL}$ is the best model among the four VaR models. Figure 6 shows the time-varying Joe-Clayton tau, left panel is the ComexGold-TocomGold portfolio, and light panel is ComexSilver-TocomSilver. It is clear that the correlation between the two assets is time-varying, especially in the ComexSilver-TocomSilver portfolio. Figure 7 also illustrates the shape of portfolio VaR based on SJC-copula at 99, 97.5 and 95% confidence levels. The left panel is the ComexGold-TocomGold portfolio, and the right panel is ComexSilver-TocomSilver.

## Conclusion

It is worthy attention that model risk in the estimation of VaR is a challenging threat for the success of any financial investments. In this article, we construct a copula based VaR-ARMAX-GJR-GARCH model. In this model, we consider the macro financial variables crude oil and US dollar/Japanese yen in a conditional mean equation, and examine the comovements and directional

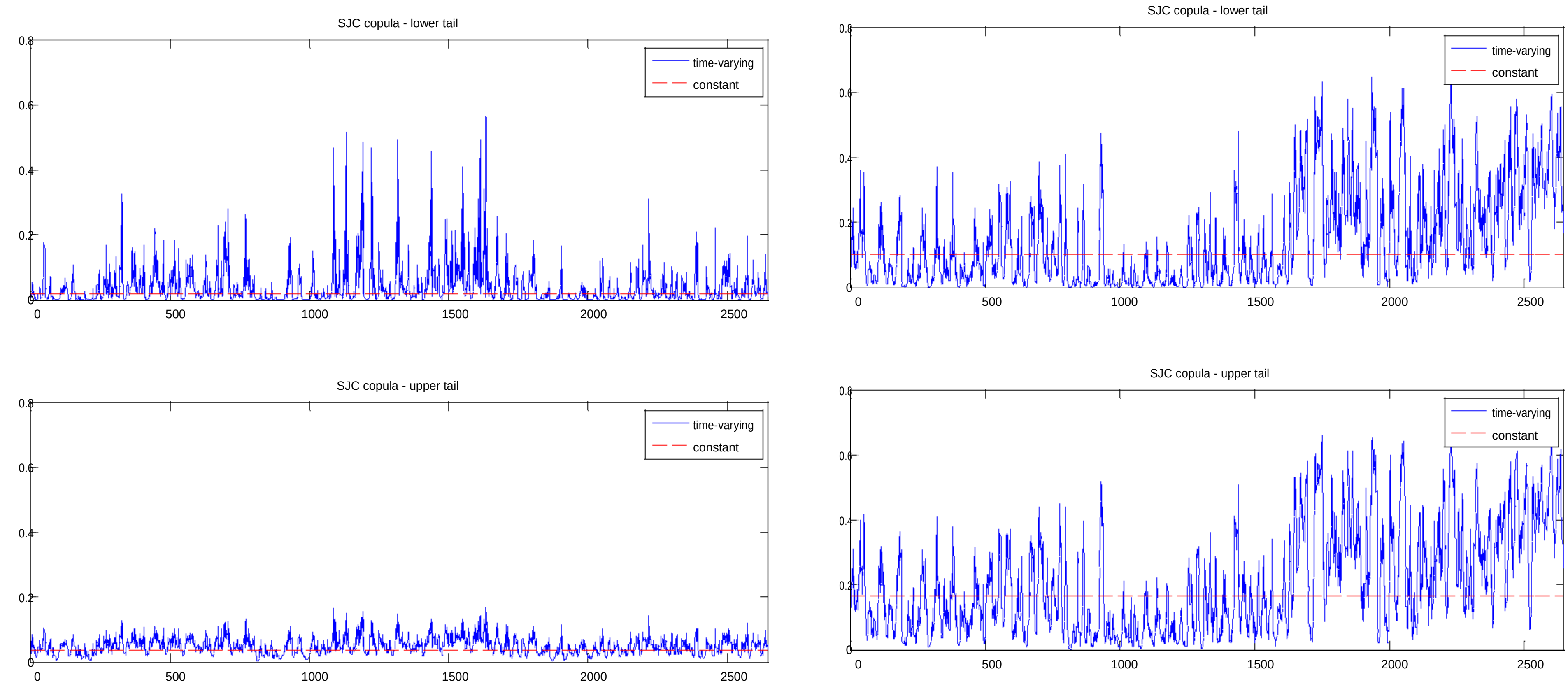


**Figure 4.** The time varying Joe-Clayton tau (left: ComexGold-TocomGold), (right: ComexSilver-TocomSilver)(before uptrend).

equation, and examine the comovements and directional relationships of strategic commodities with these variables. In addition, we employ an asymmetric GJR-GARCH to fit the conditional variance equation. In particular, we calculate the portfolio by means of some constant and time-varying copula functions. This work showed how conditional copula theory can be a very powerful tool in estimating the portfolio's VaR.

In our empirical study, we divided the data into before and during the uptrend. We examined the condition mean equation to reflect recent realities in the world commodity and financial markets. We conclude that the crude oil for the gold and silver price in Comex is significant and positive regardless of before or during the uptrend. As for US dollar/Japanese yen exchange rate, there is still no consistent result. That is to say, there is no evidence concerning the influence of the variable to gold and silver futures.

The time varying SJC copula allowing for different dependence in the tails produced the best result and reliable VaR limits. This explains why in the SJC (up tail or lower tail) can outperform other VaR models in different periods or commodities portfolios. It is also helpful for us to manage Gold or Silver portfolio risk.

Furthermore, for risk management, copula-based models provide a general framework to measure tail dependence of asset returns and,

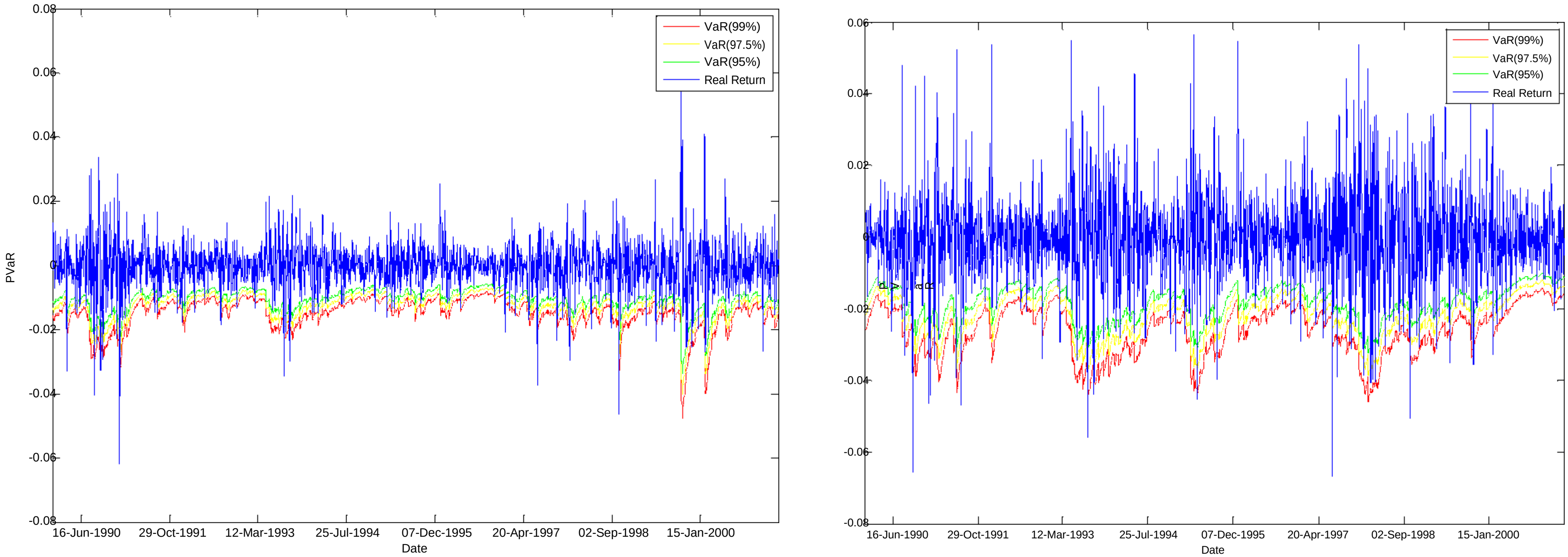


**Figure 5.** PVaR at 99, 97.5 and 95% confidence level-VAR-COV model (SJC tau) (left: ComexGold-TocomGold) (right: ComexSilver-TocomSilver) (before uptrend).

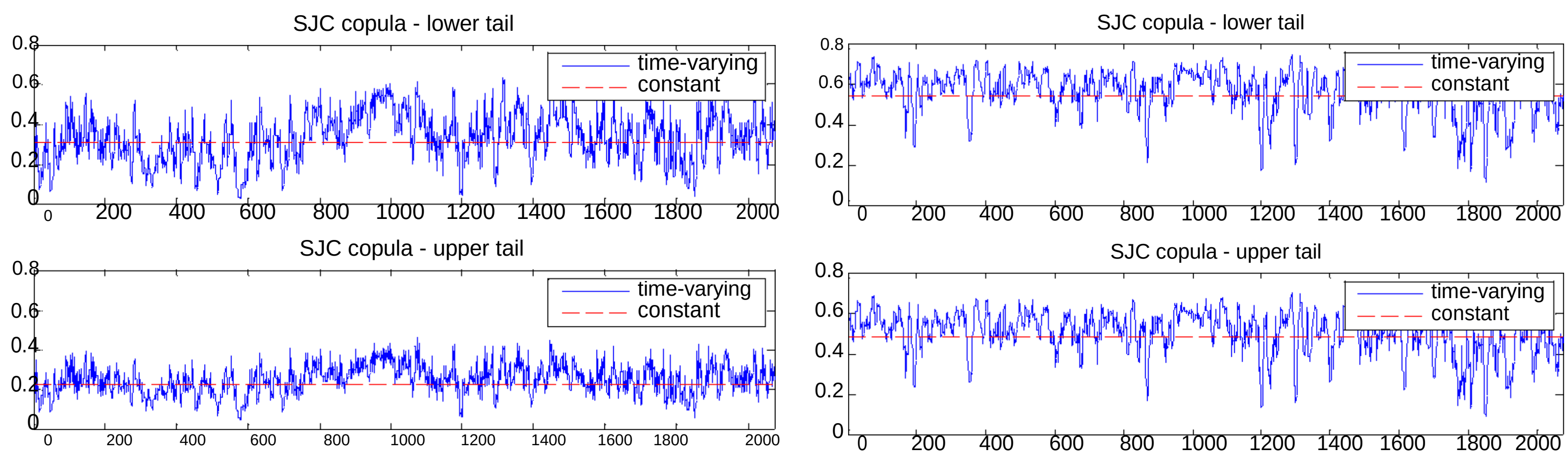


**Figure 6.** The time varying Joe-Clayton tau(left: ComexGold-TocomGold) (right:ComexSilver-TocomSilver)(during the uptrend).

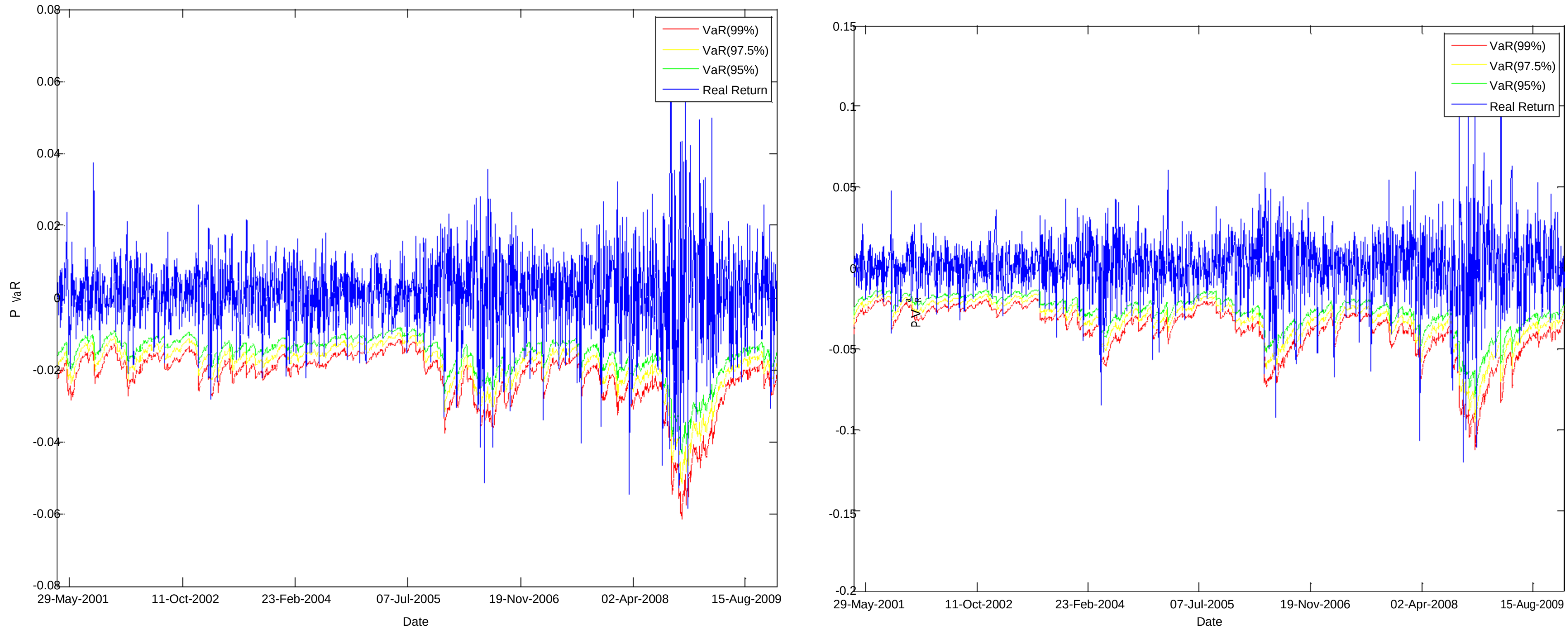


**Figure 7.** PVaR at 99, 97.5 and 95% confidence level-VAR-COV model (SJC tau)(left: ComexGold-TocomGold)(right: ComexSilver-TocomSilver) (during the uptrend).

hence, to assess more accurately portfolio risk. Indeed, since the price paths of component assets can be characterized under the copula approach, the variation of the portfolio can be measured accordingly. So, this paper shows that copula-based methods can gain insights into VaR and other risk measurements. That is to say, the model results are promising, taking into account future expectations in the development and wide-spread application of the model, permitting a greater certainty in measuring the risk of a portfolio of various financial instruments linked. There are still several ways to extend this research. First to extend this research. First, to extend this type of application to higher dimensions. Second, consider mixed copula function and use to estimate the VaR of portfolio. In last, Facing the external influences to financial instruments and financial market, consider a method for estimating the VaR of a portfolio based on copula and Extreme Value Theory (EVT).

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