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Full Length Research Paper

Impact of magnetic field on the pulsatile flow through cylindrical conduit

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This work investigates the effect of magnetic field, on the stress coefficient and the vorticity of a pulsatile flow, electrically conducting in a cylindrical conduit. The imposed magnetic field is supposed to be uniform and constant. An exact solution of the equations governing pulsatile MHD flow in a conduit has been obtained in the form of Bessel functions. The analytical study has been used to establish an expression between the Hartmann number and the stress coefficient and the vorticity variation. The numerical method is based on an implicit finite difference scheme using the Thomas algorithm and Gauss Seidel iterative method. The velocity distributions, as well as the stress coefficient and the vorticity were obtained both with and without a magnetic field. The results show that the amplitude of the vorticity increases as the Hartmann number increase. The effect of the magnetic field is significant only from Hartmann number M=5.The stress coefficient increases with the Hartmann number due to a dephasing compared to the imposed flow which increases considerably starting from Hartmann number M=10 to reach a value around 45°C.

Key words: Pulsatile flow, stress coefficient, vorticity, magnetic field, finite difference.

INTRODUCTION

The study of the fluids that exhibit oscillatory flow have various important application in nature, common examples in medical sciences included; blood flow through an artery, peristaltic food motion in the intestine, motion of urine in urethra. In astrophysics and geophysics, it is applied to the study of stellar structure, coresterrestrial and solar plasma. Vardanyan et al. [1973] have developed several theoretical models on the effect of a magnetic field on pulsatile flow. They noted that the presence of a constant and uniform magnetic field decreases the rate of flow; their work had a significant impact on biological research. Sudet al. [1974] studied the effect of the magnetic field on the sinusoidal flow of fluid in a rigid conduit. They obtained the dimensionless velocity profile. Seume and Simon [1988] present an expression for the dimensionless stress coefficient of which the amplitude and phase shift are functions which increase with the dimensionless frequency Re...With regard to the phase shift they note that it varies from 0 to 45°C. Amos and Ogulu [2002] conducted a numerical study, on the pulsatile flow in a conduit with a constriction in the presence of an external uniform magnetic field. When the magnetic field increases, the speed decreases, and the only way of circumventing this problem of speed reduction is to increase the flow pressure, which corresponds to the increase in the work load of the heart which can lead to heart attacks. Ikbaland Mandal [2008] studied the unstable response of non-Newtonian blood

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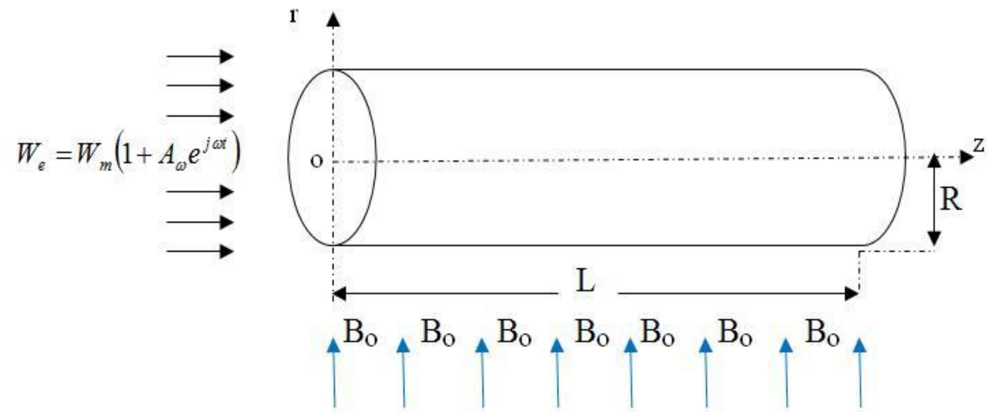

Figure 1. Geometry of the physical system.
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flow through an artery with a stenos in the presence of a magnetic field. The magnetic field is the main cause of reduced flow. Majdalani [2008] obtained the analytical solution for the evolution of the velocity profiles and the stress coefficient in the case of a pulsatile flow by Adesanya [2005]. studied the effect of couple stresses on an unsteady magneto hydrodynamic (MHD) non-Newton in flow between two parallel fixed porous plates under a uniform external magnetic field using Eyring-Power model; they reported that the flow is damped with increasing effect of couple stresses.

FORMULATION OF THE PROBLEM

Consider the pulsatile flow of an incompressible viscous fluid in a cylindrical conduit of radius *R* and length *L,* i n the presence of a transverse uniform magnetic field B_0 , (Figure 1). $B = B_0 + B_1$ represents the total magnetic field and B_1 the induced magnetic field which is neglected compared to the external magnetic field. The fluid is pulsed at the inlet of the conduit along the axiswith a periodic axial velocity W_e, pulsatior [redacted] and amplitude A given by: $W_e = W_m \left(1 + A_\omega e^{j\omega t}\right)$.

The fluid is supposed to be laminar, incompressible and all physical properties are assumed to be constant, the fluid is Newtonian and the external forces called the Lorentz electromagnetic force, due to the presence of the magnetic field are expressed after neglecting gravity for reasons of axial symmetry of the problem, in the following way: $F = J \times B$ J represents the current density due to the movement of the conductive fluid in the presence of the magnetic field, and according to Ohm's law we have: $J = \sigma(E + V \times B)$. Therefore $F = -\sigma \times B^2 \times V$. σ represents the electrical conductivity, E represents the electric field which is considered as negligible. Given all the assumptions described above and after projection of the equations on to cylindrical coordinates (r, z).

The equation governing the flow studied is reduced to:

Equation of motion

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{\sigma B^2 w}{\rho} \quad (1)$$

With, $\frac{\partial P}{\partial z} = -\left(A_0 + A_1 e^{i\omega t}\right)$

MATHEMATICAL ANALYSIS

Using the following dimensionless variables:

$$r^* = \frac{r}{R}, \quad z^* = \frac{z}{R},$$
$$t^* = \frac{tW_m}{R}, \quad P^* = \frac{P}{\rho W_m^2}, \quad U^* = \frac{u}{W_m}, \quad W^* = \frac{w}{W_m}.$$

Where (*U, W), (r, z), t, P* are the velocity coordinate, cylindrical coordinates, time, and the pressure.

R_e is the Reynolds number and M the Hartmann number, defined as follows: $R_e = \frac{\rho W_m R}{\mu}$, $M^2 = \frac{\sigma B_0^2 R^2}{\mu}$

Therefore, the equation of motion, governing the considered phenomenon can be written as follows:

$$\frac{\partial W^*}{\partial t^*} + U^*\frac{\partial W^*}{\partial r^*} + W^*\frac{\partial W^*}{\partial z^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{R_e}\left[\frac{\partial^2 W^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial W^*}{\partial r^*} + \frac{\partial^2 W^*}{\partial z^{*2}}\right] - \frac{M^2}{R_e}W^* \quad (2)$$

Boundary conditions

- At the entrance of the conduit z = 0

$W(r,0,t) = (1 + A_\omega e^{j\omega t})$

$U(r,0,t) = 0$

- At the exit z = *L/R* $\frac{\partial W}{\partial z}\left(\frac{L}{R}, r, t\right) = \frac{\partial U}{\partial z}\left(\frac{L}{R}, r, t\right) = 0$ on the axis r=0 $\frac{\partial W}{\partial r}(0, z, t) = U(0, z, t) = 0$
- At the wall of the conduit r = 1 $W(1, z, t) = U(1, z, t) = 0$

Analytical study

In addition to the previous simplifying assumptions, we suppose that the flow occurs only in the axial direction. Under these conditions, the conservation equation governing the pulsated flow is as follows:

$$\frac{\partial W}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{R_e}\left[\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right] - \frac{M^2}{R_e}W \quad (3)$$

The velocity field is expressed as follows:

$W(r, t) = W_0(r) + W_1(r, t)$

$W_0(r)$ and $W_1(r, t)$ represent respectively the stationary and non-stationary part of the velocity.

Study of the steady state

The dimensionless equation to be solved in the stationary case is:

$$\frac{d^2W}{dr^2} + \frac{1}{r}\frac{dW}{dr} - M^2 W = -A_0 R_e \quad (4)$$

The solution of the homogeneous equation is written in the form:

$$W_0(r) = C_1 I_0(M.r) + C_2 K_0(M.r) \quad (5)$$

With: $\frac{A_0 R_e}{M^2}$ as a particular solution and solution and I_0, K_0 are modified Bessel functions of the first and the second kind of order 0. C_1, C_2 are constants calculated from the previous boundary conditions. We then find:

$$W_0(r) = \frac{A_0 R_e}{M^2}\left(1 - \frac{I_0(M.r)}{I_0(M)}\right) \quad (6)$$

Study of the not stationary state

To characterize the oscillatory flow, we include the dimensionless frequency $R_{e\omega}$ and we use dimensionless variables of time $t^* = \omega t$, where: $R_{e\omega} = \frac{\rho R_2 \omega}{\mu}$ is equal to the square of the Womersley number $R_{e\omega} = \alpha^2$. In this case it is the gradient of pressure which generates the flow. Under these conditions, the dimensionless velocity profile can be put in the form: $W_1(r,t) = \mathrm{Re}(f(r)e^{it})$

The dimensionless equation to solve in the non-stationary case is:

$$\frac{d^2}{dr^2}f + \frac{1}{r}\frac{d}{dr}f - (M^2 + i\alpha^2)f = -A_1\alpha^2$$

The solution of the equation without second term $f_s(r)$ is written in the form:

$$f_s(r) = C_1 I_0(\sqrt{M^2 + i\alpha^2}.r) + C_2 K_0(\sqrt{M^2 + i\alpha^2}.r)$$

with: $f_p(r) = \frac{A_1\alpha^2}{M^2 + i\alpha^2}$ as the particular solution.

The constant C_2 is set equal to zero because the velocity cannot be infinite at $r = 0$. C_1, is determined by the condition of zero velocity at the wall expressed by

$$f_g(1) = 0$$

Such as:

$$f_g(r) = f_s(r) + f_p(r)$$

The general solution is as follows:

$$f_a(r) = -\frac{A_1\alpha^2}{M^2 + i\alpha^2}\left(\frac{I_0(\sqrt{M^2 + i\alpha^2}.r)}{I_0(\sqrt{M^2 + i\alpha^2})} - 1\right)$$

Letting $\beta = \sqrt{M^2 + i\alpha^2}$

We obtain the expression of the dimensionless velocity:

$$W_1(r, t) = -\frac{A_1\alpha^2}{\beta^2}\left(\frac{I_0(\beta.r)}{I_0(\beta)} - 1\right)e^{it}$$

The value of A_1 is not known hence, we use the flow velocity $W_m = \mathrm{Re}(W_{max} e^{it})$, Such as:

$$W_{max} e^{it} = 2\int_0^1 f_g(r) r e^{it} \omega R dr,$$

$$W_{max} = -\frac{A_1\alpha^2}{\beta^2}\left(\frac{\omega R\, 2 I_1(\beta.)}{\beta I_0(\beta)} - 1\right)$$

We deduce:

$$A_1 = -\frac{W_{max} P_2}{\alpha^2 \omega R}\left(\frac{1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right) = -\frac{R_{e\,max} P_2}{2\alpha^4}\left(\frac{1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)$$

With

$$R_{e\,max} = \frac{2\alpha^2 W_{max}}{\omega R}$$

Where:

$$W_1(r,t) = \frac{R_{e\,max}}{2\alpha^2}\left(\frac{\frac{I_0(\beta.r)}{I_0(\beta)} - 1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)e^{it} \quad (7)$$

This is the same expression found by Bouvier [2005] for the velocity profile of an oscillatory flow in the absence of a magnetic field.

Study of the pulsatile state

The expression of the dimensionless velocity pulse is the superposition of the stationary component and the oscillatory component.

$$W(r,t) = W_0(r) + W_1(r,t) = \frac{A_0 R_e}{M^2}\left(1 - \frac{I_0(M.r)}{I_0(M)}\right) + \frac{R_{e\,max}}{2\alpha^2}\left(\frac{\frac{I_0(\beta.r)}{I_0(\beta)} - 1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)e^{it}$$

The dimensionless stress coefficient at the wall τ is calculated from the previous expression:

$$\tau = \tau_s + \tau_{osc} \quad \tau = -\left.\frac{\partial W}{\partial r}\right|_{r=1} = \frac{A_0 R_e}{M}\frac{I_1(M)}{I_0(M)} - \frac{\beta R_{e\,max}}{2\alpha^2}\left(\frac{\frac{I_1(\beta)}{I_0(\beta)}}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)e^{it}$$

Calculation of the stress coefficient

Direct calculation of the stress coefficient is not possible from the equations governing the flow, which led us to use an indirect method involving the balance of forces in a volume element of fluid commonly used in such studies. If we considerthe flow ofan incompressible fluidwith a flow velocity in a cylindrical conduit in the presence of a magnetic field, we can write for a given volume element, the equation of balance forces acting on it as follows:

$$\pi R^2 \rho \frac{\partial W_m}{\partial t} + \pi R^2 \frac{\partial P}{\partial z} = -2\pi R \tau_p - \pi R^2 \sigma B^2 W_m \quad (8)$$

From Equation 12, the expression for the wall stress is given as follows:

$$\tau_p = -\frac{R}{2}\left(\frac{\partial P}{\partial z} + \rho\frac{\partial W_m}{\partial t} + \sigma B^2 W_m\right) \quad (9)$$

Where:

$$R_e = \frac{\rho W_m R}{\mu},\ M^2 = \frac{\sigma B_0^2 R^2}{\mu}.$$

The expression of the stress coefficient is then:

$$C_f = -\frac{R}{\rho W_m^2}\left(\frac{\partial P}{\partial z} + \rho\frac{\partial W_m}{\partial t}\right) - \frac{M^2}{R_e} \quad (10)$$

Equation 8 can be written using the previous dimensionless variables, adding the dimensionless variable $C_{f\,normalized}$ called normalized stress coefficient as follows:

$$C_{f\,normalized} = -\frac{i\alpha^2}{8}\left(1 + \frac{1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)$$

With:

$$C_{f\,normalized} = \frac{\tau_p}{\tau_s}, \text{ and } \tau_s = \mu 8 \frac{W_m}{2R}$$

We use the flow velocity $W_m = \mathrm{Re}(W_{max} e^{it})$, and the equation, is written in the oscillatory regime as follows:

$$iR\omega W_{max} e^{it} + R\omega^2 \frac{\partial P}{\partial z} = -\frac{\mu\omega 8}{\rho R} C_f W_{max} e^{it} - \frac{\sigma B^2}{\rho} R\omega W_{max} e^{it},$$

where: $M^2 = \frac{\sigma B_0^2 R^2}{\mu}$, $\alpha^2 = \frac{\rho R^2 \omega}{\mu}$

The previous expression becomes:

$$i\alpha^2 + \frac{\alpha^2}{W_{max} e^{it}}\frac{\partial P}{\partial z} + M^2 = -8C_{f\,normalized}$$

$$C_{f\,normalized} = -\frac{M^2}{8} - \frac{i\alpha^2}{8}\left(1 - \frac{\frac{i\beta^2}{\alpha^2}}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right) \quad (11)$$

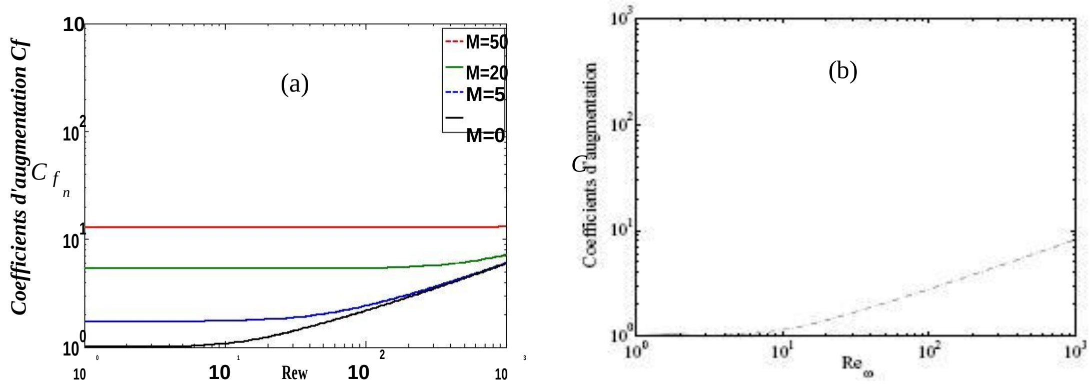


Figure 2. The $C_{f\,normalized}$ as a function of Reω for different Hartmann numbers (a) Current study (b) Seume and Simon (1988) result.

In the absence of the magnetic field

$$M^2 = 0, and \beta^2 = i\alpha^2$$

We find the same expression given by Seume and Simon [1988]:

$$C_f = -\frac{i\alpha^2}{8}\left(1 + \frac{1}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)$$

Numerical study

The discretization of the above equations using a finite difference implicit method used by Ghezal [2007] leads to a system of algebraic equations which were solved using an iterative Gauss-Seidel technique with a relaxation coefficient.

Figure 2a shows that the maximum amplitude of the $C_{f\,normalized}$ in the absence of magnetic field *(M= 0)*, increases with frequency to about 10times its value in the case of steady flow. These results are consistent with those obtained by Seume and Simon [1988], Figure (2b). However there was a slight increase for low frequencies. This can be justified by the contribution of the second component of the velocity. The frequency effect is appreciable for frequencies Re_ω between 100 and 150, which corresponds to the initiation of the ring effect.

RESULTS AND DISCUSSION

The $C_{f\,normalized}$ is approximately equal to 1 for low frequencies Re_ω <100, which corresponds to the case of steady flow. This result is found by the limited development of the expression (15) in the absence of the magnetic field and for $Re_\omega \to 0$.In fact; the limited development near 0 of the function $I_1(\beta)$: which is given by: $I_1(\beta) = \frac{\beta}{2} + \frac{\beta^3}{16}$. The expression $C_{f\,normalized}$ is then:

$$C_{f\,normalized} = -\frac{i\alpha^2}{8}\left(1 + \frac{1}{\frac{\beta + \frac{\beta^3}{8}}{\beta} - 1}\right) = \frac{i\alpha^2}{8}\left(1 + \frac{8}{\beta^2}\right) = 1 + \frac{i\,Re_\omega}{8}$$

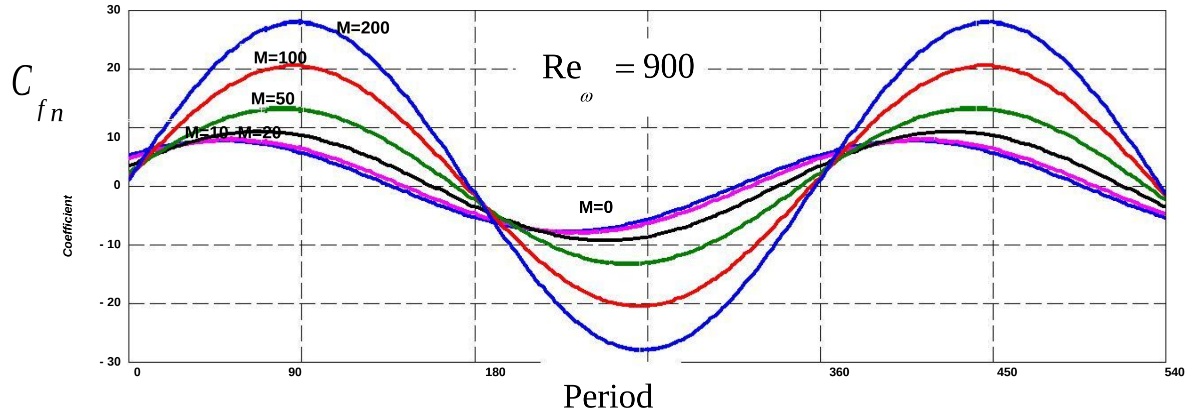
This gives for,

$$Re_\omega = 0\, C_{f\,normalized} = 1$$

In the presence of the magnetic field $C_{f\,normalized}$ increases with the Hartmann number for a given frequency, the maximum amplitude varies as a function of the frequency from Re_ω >100.

Another important result is that the variation of $C_{f\,normalized}$ in the presence of the magnetic field is less dependent on frequency as the Hartmann number increases. The variation becomes totally independent of frequency for values of Hartmann M>20.The expression of the $C_{f\,normalized}$ as a function of time is given as follows:

$$C_{f\,normalized}(t) = -\frac{M^2}{8} - \frac{i\alpha^2}{8}\left(1 - \frac{\frac{i\beta^2}{\alpha^2}}{\frac{2I_1(\beta.)}{\beta I_0(\beta)} - 1}\right)e^{it}$$



Figur 3. $C_{f_{normalized}}$ as a function of time for different Hartmann numbers for a high value of Re_ω.

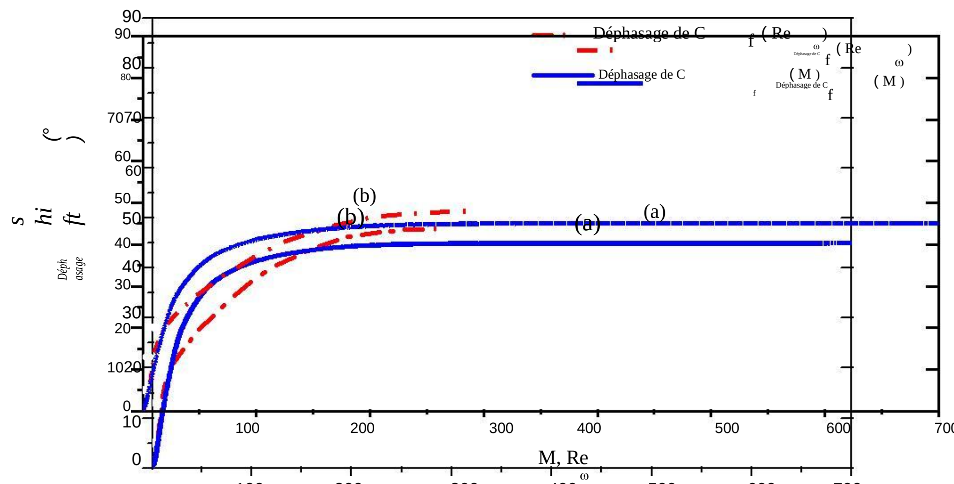


Figure 4. The variation of the phase shift of $M, C_{f_{normalized}} Re_\omega$ as a function of Hartmann numbers and the frequency Reω.

The variation of this magnitude for different Hartmann numbers is given in Figures 3 and 4. From this curve, it is found that $C_{f\,normalized}$ is sinusoidal and increases with Hartmann number without dephasing at low values of Re_ω.

The influence of magnetic field on the dephasing (phase shift) of the $C_{f\,normalized}$ is of the same nature as that of the frequency, these results are in good agreement with those found by Bouvier [2005].

Influence of Hartmann number on the stress coefficient

From (14), it is found that the stress coefficient is sinusoidal and it increases with the increase of the Hartmann number causing a dephasing which greatly increases from *M = 10* to reach a limit value around 45 °. It should be noted that these results are consistent with those obtained analytically and can therefore be used in the validation of our numerical code.

CALCULATION OF THE VORTICITY

Dimensionless vorticity is given by:

$$\Omega = -\frac{dW}{dr} = -\frac{dW_0(r)}{dr} - \frac{dW_1(r,t)}{\partial r}$$

$$W = \frac{AR_0}{M^2}\left(1 - \frac{I_0(M.r)}{I_0(M)}\right) - \frac{A\alpha^2}{\beta^2}\left(\frac{I_0(\beta.r)}{I_0(\beta)} - 1\right)e^{it}$$

After simplification, the final expression of the vorticity is

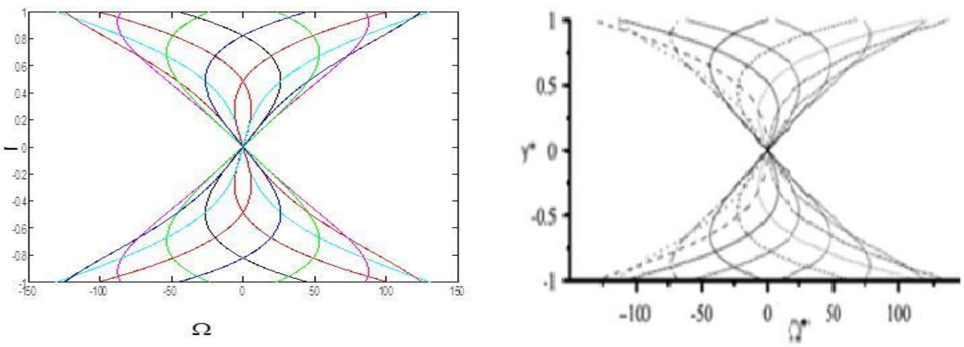
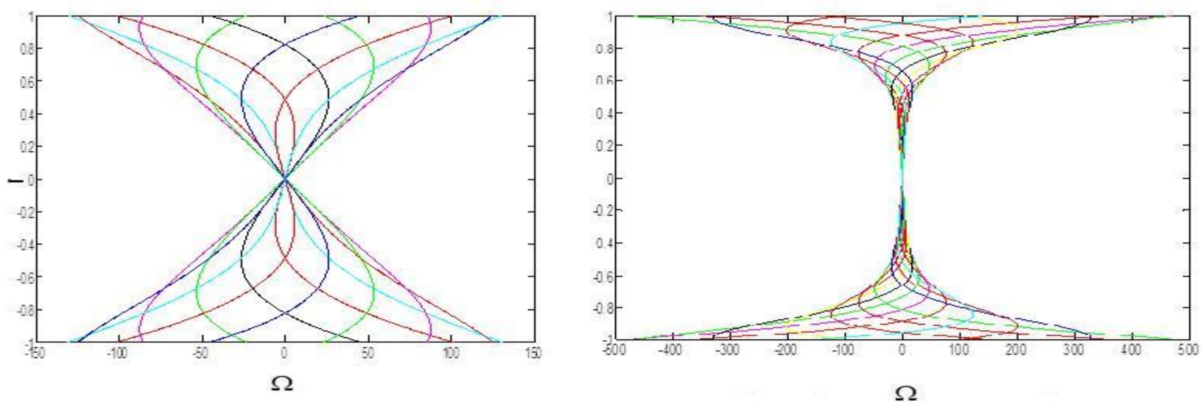


Figure 5. Radial profile of the vorticity (*M* → *0,* Reω=10, *A=50).* (a) Current study, (b) Results of Majdalani [2008].



(a) (b)

Figure 6. Radial profile of the vorticity (Reω=10,*A=50)*(a) *M* → *0*, (b) *M =5.*

given as follows:

$$\Omega = \frac{A_{0e} R}{M} \frac{I_1(Mr)}{I_0(M)} + \frac{A\alpha^2}{\beta}\left(\frac{I_1(\beta . r)}{\beta I_0(\beta)}\right) e^{it}$$

Where: A_0 represents the amplitude of the stationary part. A represents the amplitude of the oscillatory part. The vorticity is plotted using MATLAB as a function of radius r for different values of Hartmann number and frequency for different phases and for different ratios of amplitudes.

For a low Hartmann number and high amplitudes (A = 20, A = 50)

The influence of the amplitude is shown in Figure 5. The results indicate that the values of the vorticity are higher than the amplitude values. By comparing our results with those found by Majdalani [2008], on the purely oscillatory flow in the absence of magnetic field, for an amplitude A = 50 and frequency $\mathrm{Re}_{\omega} = 10$, we note that they are in good agreement.

For a high Hartmann number and high amplitudes (A=50)

Figure 6 Shows the effect of magnetic field on the radial profile of vorticity. We note that the magnitude of the vortices is higher when the Hartmann number is high. It should be noted that the effect of the magnetic field is significant only from *M=5*. It is noted that the vorticity takes negative values for certain phases for different amplitude values indicating the presence of a return flow.

Conclusions

In this work we have calculated the stress coefficient based on the one-dimensional case used in some works, because the calculation in the case of bi-dimensional unsteady flow presents some difficulties due to the change of sign of the velocity value over a cycle. This also allows the detection of the phase shift between the stress coefficient and the flow velocity without much difficulty. The analytical study has permitted us to establish expressions reflecting the velocity profile, the variation of the stress coefficient and vorticity as a function of Hartmann number. It has also shown that the stress coefficient has a sinusoidal aspect and it increases with the increase of the Hartmann number causing a phase shift which considerably increases from M = 10 to reach a limit around 45°C, and the amplitude of the vorticity are higher when the Hartmann number is high. It should be noted that the effect of magnetic field on the vorticity is visible only from M = 5.

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